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URBAN SPRAWL INDICATOR BASED ON WEIGHTED ACCESSIBILITY MEASURE

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ABSTRACT

Significant progress has been made in order to develop methodologies aiming at measuring urban sprawl. However, most of the current methodologies put great emphasis on density and urban form measurement, while accessibility receives little attention. Methodologies commonly employed to measure sprawl do not grasp details of intra-urban levels, including street network configuration and accurate location of urban activities, such as residential and job location. Hence, accessibility remains almost unaddressed in urban sprawl measurement studies. Measuring accessibility is important because it enables us to tackle the most obvious negative effects of urban sprawl, such as the increase in commuting distances. In this paper we propose an urban sprawl indicator, which is based on weighted accessibility measure. It attempts to measure spatial mismatch between residential and employment location. This paper aims at bringing urban sprawl measurement to a network and configurational perspective, which is helpful to measure accessibility. For this purpose, we describe the urban system as a graph where nodes represent small portions of urban built forms and lines represent the street segments connecting them. The method was applied to Torres, a small town in Brazil, where some hypothetical scenarios were carried out in order to compare the results. The results indicate that the proposed method has potential to be used in comparative studies about urban sprawl, since it enables us to grasp different mismatching degrees between residential and job location. We conclude that commuting distances are influenced not only by spatial mismatch between activities but also by its accessibility in relation to the whole system.

KEYWORDS

Urban sprawl measurement, graph-based approach, spatial configuration, accessibility measure, urban performance indicator

1. INTRODUCTION

Urban sprawl has been often characterised as low-density and monofunctional urban development. Moreover, it has been often related to negative effects, such as increasing commuting distances, intensive use of individual vehicles, and greater land consumption, albeit this kind of correlation is yet to be proven with more empirical research. Over the past few decades, there has been increasing concern about this phenomenon and its impacts. Currently, there is general agreement that it should be monitored in order to better guide urban development policies.

Recently, several researchers have developed quantitative indicators and indexes to compare the sprawl degrees of cities or to monitor the phenomenon in a single city over the time. Despite the great effort that has been made, much of the existing work has excessively focused on density patterns and general urban form that characterises urban sprawl. Meanwhile, aspects such as accessibility and details in intra-urban scale – streets network configuration and activities distribution – have received less attention. Such aspects are precisely the ones that should be more explored in quantitative work because they are strongly related with sprawl effects and raise a more accurate debate on urban performance. We have to focus on the fact that what makes sprawl undesirable is its negative effects and not its urban form. Actually, the correlations between urban morphology and urban real effects are still yet to be proven, and, because of this, we urgently need to improve our methods to reach urban performance.

The key argument in this article is that we need to develop urban sprawl measurement tools towards indicators that are more strongly related to impacts, such as accessibility. In this paper, we make an effort to bring the discussion on urban sprawl measurement into a configurational approach. Little attention has been given to particularities of intra-urban scale in sprawl literature. Nevertheless, highly developed methods can be found in urban configurational studies. Therefore, we suggest an accessibility-based indicator, which is derived from a configurational and systemic approach. Such methodology is capable of providing a better description system of morphological and configurational properties of the urban spatial structure. A more accurate description, in turn, enables more accurate and sophisticated strategies to measure sprawl.

Another important point to consider is that most of the current methods for measuring sprawl use aggregated data that poorly captures configurational and fine-grained issues. Fortunately, it is becoming increasingly easier to find refined urban data and computational tools that perform measures in intra-urban level, so it is worth making use of this.

Next section provides a brief review of some urban sprawl measures research recently developed. In particular, we attempt to verify how accessibility has been approached in those studies and its main shortcomings. The third section presents our proposal of a new urban sprawl indicator, which is based on a weighted accessibility measure. This section also provides background on urban configurational studies and a pilot study. The fifth section presents a case study where the proposed method was applied. Finally, the last section provides final consideration.

2. URBAN SPRAWL MEASUREMENT

The first challenge one faces when dealing with urban sprawl measurement is its unclear conceptual definition. Sprawl has become an umbrella term covering many forms of development (Longley, Batty and Chin, 2002). Although there is no consensus on a definition for urban sprawl it is widely accepted that it refers to low-density development and to high reliance on the automobile, because people live too far from workplaces, study places and shopping places (Jaret et al., 2005).

In the last few decades, sprawl has been conceptualised as a matter of degree, not an absolute form (Chin, 2002), as well as a multidimensional phenomenon that requires a different set of measures for each dimension. Several scholars (Galster et al., 2001; Ewing, Pendall and Chen, 2002; Ewin and Hamidi, 2014; Bertaud and Malpezzi, 2003; Ribeiro and Holanda, 2006; Torrens and Alberti, 2000; Torrens, 2008; Frenkel and Ashkenazi, 2008) have developed quantitative indexes for measuring sprawl degree and comparing cities. Such methods play an important role in sprawl debate due to the promotion of a more rigorous definition of the phenomenon. In this section, we highlight some relevant aspects of those methodologies for measuring sprawl, in particular, some gaps concerning accessibility assessment.

2.1 LITERATURE REVIEW

Galster et al. (2001) defined sprawl as a condition that is represented by low values on one of the eight dimensions of the phenomenon: density, continuity, concentration, clustering, nuclearity, mixed uses, proximity. The researchers developed operational measures for each dimension.

According to Jaret et al. (2005), this work is a conceptual and methodological breakthrough in urban sprawl measurement.

Ewing, Pendall and Chen (2002) focus on sprawl impacts rather than sprawl characteristics and urban form. The authors propose a composite index based on the four main factors that are measurable: a) density; b) mix of homes, jobs and services; c) strength of activity centres and downtowns; and d) accessibility of the street network. They apply those measures to 83 metropolitan areas in the United States and relate then to quality of life indicators, such as time of commuting, air pollution and traffic jam. Later, the research was updated with more recent data (Ewing and Hamidi, 2014). Their accessibility measures are related only to block size, overlooking configurational issues.

Bertaud and Malpezzi (2003) focus on spatial distribution of the population. They examine density gradients and propose an alternative measure of urban dispersion by synthesising densities and distances to the downtown into a single index. Both methods are related to distance between population and CDB (Central Business District). The measurement methods are applied to almost 50 cities all over the world. It was one of the first efforts to study urban form systematically around the world. Later, Ribeiro and Holanda (2006) have included seven Brazilian cities to the sample. The method proposed by Bertaud and Malpezzi (2003) is interesting because it faces the problem of accessibility between the population and the CDB, where the job opportunities are located. The problem is that it assumes that all the employments are in the CDB, which means a monocentric pattern is assumed. However, as cities expand, they tend to assume a polycentric pattern (Longley, Batty and Chin, 2002; Louf and Barthelemy, 2013) and the employments are most likely to be dispersed (Song, 1994). Accordingly, population distribution can be better explained by a model that accounts for distance to all employment rather than just to employment in CDB (Song, 1994; Anas, Arnott and Small 1998).

Frenkel and Ashkenazi (2008) make use of measures from different disciplines to carry out a study about sprawl in seventy-eight urban settlements in Israel. The thirteen chosen variables are related to three dimensions of urban sprawl: density, scatter, and mixture of land uses. The measures were weighted into one integrated sprawl index in order to compare cities. The authors attempt to measure urban sprawl from a multidimensional perspective. However, they leave out accessibility, which is an important dimension of the phenomenon.

Unlike the authors mentioned above, who analyse a great number of cities, Torrens (2008) focus on only one city: Austin. The study embraces forty-two measures, which attempts to embrace the full range of sprawl characteristics: urban growth, density, social, activity-space, fragmentation, decentralisation, and accessibility. One of the main contributions of this study is to achieve multiple scales, including the intra-urban level. The author examines accessibility patterns: to the CDB, to major employers, to schools, to other educational opportunities, and to locally unwanted land-uses.

2.2 CRITICAL ANALYSIS

Most of the studies mentioned above consider multiple dimension of sprawl, including density, general urban form, mix of uses and accessibility. Here we highlight the main shortcomings:

- Density and urban form overemphasised, effects overlooked
- Little attention has been paid to intra-urban scale. The research we have analysed deals almost exclusively with aggregated data, usually in the metropolitan scale. Torrens (2008) is an exception.
- The urban form is assumed to be monocentric in almost all of the related studies. However, many authors agree that the real nature of cities is more polycentric instead of monocentric (Song, 1994; Longley, Batty and Chin, 2002; Louf and Barthelemy, 2013).
- Measures are not systemic and do not consider spatial relationship among activities
- Accessibility poorly explored.

As we could see, density is the most studied dimension of sprawl. Measuring and modelling the spatial distribution of residential density has been attempted in several ways. However, according to Ewing, Pendall and Chen (2002) density should not be overemphasised. Researchers should pay more attention on how urban sprawl affects the urban environment.

Accessibility, in turn, is a key point in urban performance debate since it is strongly related to efficiency, equity and sustainability (Bertuglia, 1994; Netto and Krafta, 2009). Low accessibility can be considered one of the most undesirable aspects of urban sprawl, since residential areas may be too far from jobs and services (Ewing, 1997). Consequently, it can lead to more fuel consumption and automobile dependence. Despite all the possible bad impact in urban performance, accessibility has received little attention in sprawl measurement studies. Although accessibility is regarded as an important dimension of sprawl it is not always included in an operational way.

The literature review lead us to the conclusion that one of the main shortcomings of current methodologies is that urban morphology in intra-urban level is poorly described. As we will see in Section 3, a network perspective can contribute with a more precise description of the activities distribution and their spatial relationship, leading us to operationalize accessibility.

3. BRINGING SPRAWL MEASUREMENT INTO A NETWORK PERSPECTIVE

Urban sprawl exhibits poor accessibility, first because residents are often distant from urban opportunities, such as work, shopping, and entertainment. Second, because opportunities are themselves far from each other (Torrens and Alberti, 2000). On that account, measuring spatial mismatch among urban functions seems to be a good urban sprawl indicator. However, we need to handle with urban spatial features in intra-urban scale to give a step further in the direction of impacts assessment. The main features to accurately assess accessibility are the street network and the spatial distribution of residential and non-residential areas.

A traditional approach would define a unit of study, such as neighbourhoods or census tracts, and compare number of households and employments, or measure distance from residential areas to the CDB. Much of the sprawl measurement literature is based on such type of analysis. The main assumption in this type of study is that each unit is independent of the others. In such approach, accessibility would be assessed by measuring distances in straight-line from each unit of study to a monocentric downtown area.

However, we already have more sophisticated methods to measure accessibility, under a configurational and systemic approach. Besides, we already have methods to represent cities in a more detailed way, describing street network and activities distribution. Urban spatial configurational studies have tackled the intra-urban scale by using a detailed description of urban configurations. Such approach regards cities as networks of public spaces (Hillier and Hanson, 1984) and urban built forms, which host urban activities (Krafta, 1994). In the network analytic framework, the basic unit is a pair of units tied by some kind of relationship. This way, one can measure the interaction between all possible pairs of units.

Several graph-based methods for describing the urban spatial system and for operationalising measures have been suggested in the literature (Hillier and Hanson, 1984; Krüger, 1990; Krafta, 1994; Batty, 2004; Porta, Crucciti and Latora, 2006, Figueiredo, 2015). However, sprawl literature has rarely incorporated this kind of study.

In this article, we tackle urban sprawl measurement from a network perspective. We attempt to develop a measure from a systemic and configurational point of view because it seems to be most appropriate to describe and assess accessibility accurately.

4. URBAN SPRAWL INDICATOR BASED ON WEIGHTED ACCESSIBILITY

This section presents an attempt to deal with accessibility in an operational way. The first step was to figure out a good description system, then the calculation method. Finally, a pilot study is also provided.

4.1 DESCRIPTIVE SYSTEM

A graph-based methodology is proposed to describe the urban spatial structure because it enables disaggregation of urban system in small portions. Although urban sprawl is generally characterised in metropolitan scale, we argue that urban sprawl must be assessed also by its characteristics in intra-urban level. Details such as street network configuration and jobs and household locations can be considered as important as general urban form and general population distribution. Scholars have developed several descriptive systems which are graph-based. Figure 1 summarises the main ways in which streets network can be represented as maps and their corresponding graphs.

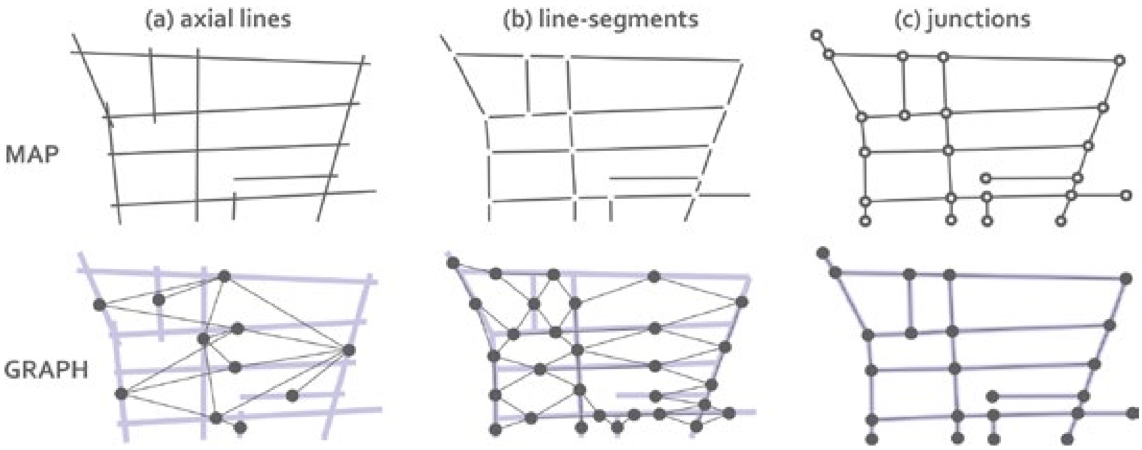

Figure 1 - Different descriptive system and its corresponding graphs.

Krafta (1994) proposes an urban graph which describes public open space and built forms. This way, elementary portions of space are represented by dots, and the connectivity among them is represented by lines. Urban built forms are, actually, small portions of the city and can host attributes of residential use, non-residential use or both.

The descriptive system we adopt is an urban graph which is very similar to the schema "c" showed in Figure 1. However, instead of mere junctions, the nodes describe small portions of the city containing urban built forms, following the descriptive system suggested by Krafta (1994). Each node contains a set of urban built forms so that nodes can be loaded with attributes about the number of households or quantity of jobs, for instance. One advantage of this description system is that it enables us to have an accurate description of the distribution of urban activities and how they connect to each other.

Besides accurately describing urban activities location, the main advantage of this description system is that it enables to measure network distances between activities since Euclidean distances are preserved in the graph representation. It can be regarded as a primal approach (Batty,2004; Porta, Crucciti and Latora, 2006), which means that distances in the map are the same as in the graph. Graph-based measures are often based on topological distances, but to deal with sprawl measurement it seems to be better using Euclidean distances. Another possible way to measure distances could be by temporal distance, or in other words, commuting time. In this paper, we assume that the proposed measurement method do not consider any transportation facilities, which in turn could change considerably the commuting time. The model considers only the configuration of the streets network and the distribution of activities.

Therefore, the descriptive system adopted depicts the city as a system. Links between activities are highlighted, making it possible to develop a systemic indicator based on graph-based measures.

4.2 ACCESSIBILITY MEASURE

A common way to measure accessibility is to produce a composite index of accessibility from one place to all others (Batty, 2009). Since accessibility implies in some measure of proximity (Ingram, 1971), and proximity can be measured by distances, a sum of distances can be used to measure accessibility. Torrens and Alberti (2000) argue that a gravity-based measure is one of the possible methods that can be used to assess accessibility in sprawl measurement context.

In this paper, we adopt a method based on gravitational models (Haynes and Fotheringham, 1984; Torrens and Alberti, 2000). In such approach, we have the following components: the capability of a point in generating trips; the attractiveness of a destination; the cost or distance among all pairs of origin and destination; and some mechanism of weighting to discourage long trips. We attempt to adjust this classical measure to the sprawl measuring concerns. Therefore, we consider the capability of generating trips as the number of households and the attractiveness of a destination as the number of workplaces. Distances are measured by metric network distances of the shortest path among the urban activities. We do not consider any mechanism to discourage long trips because the intention is exactly to assess the increasing distances between activities.

Hence, the proposed measure of weighted accessibility (Aij) in a given point is a sum of distances (dij) weighted by origin (Oi) and destination (Dj) attributes.

$$A_i = \sum_{j=1}^{n} \left(d_{ij} O_i D_j \right)$$

Equation 1 - Weighted Accessibility measure for each node

The algorithm does not consider distances from one point to all others. It computes only distances between pairs of nodes that contain complementary activities, like households and workplaces. To use the gravitational model language we will call the households as origin points and the workplaces as destination points. The algorithm identifies all the origin-destination pairs of nodes before starting the calculation. The first step to measure the weighted accessibility of a given point is to detect all the nodes which contain destination attributes. The second step is to measure the distance of the minimal path between each origin-destination pair and multiply it by the attributes of the origin point and of the destination point. Then, sum up all.

This measure can be read as the potential that each point has to cause undesirable consequences to the city. The results are computed for each node, so it can be used as a local indicator of the potential that each part of the city has to cause undesirable effects related to increasing distances. For example, if you build a huge residential area far from all the urban facilities and jobs, it will have undesirable effects powered by the number of people living there, increasing vehicular dependence and fuel consumption.

Nevertheless, our focus in this paper is to propose an urban sprawl indicator aggregated for a whole region. The purpose of this indicator is to compare different urban settlements from the point of view of spatial mismatching between residences and employments location. The proposed indicator basically consists of a weighted average of all values obtained for weighted accessibility measurements in a particular urban graph. Since cities are very different in size and in their urban configuration characteristics, such indicator should capture all those aspects. Even tough it is an aggregated indicator it can still be regarded as a systemic measure since it is obtained from a graph-based measure. In other words, it considers the spatial connection among different parts of the city.

The Sprawl Indicator (I^{sprawl}), showed in Equation 2, is the sum of weighted accessibility of each node divided by the total number of origin attributes (O_{total}) and the total number of destination attributes (D_{total}). The higher the value found for the Sprawl Indicator the higher the distances between households and employments.

$$I^{sprawl} = \frac{\sum A_i}{(O_{total} D_{total})}$$

Equation 2 - Sprawl Indicator based on Weighted Accessibility

4.3 PILOT STUDY

The Sprawl Indicator was applied to some sets of theoretical models that represent different urban configuration patterns, concerning street network and spatial distribution of employment and residences (Figure 1). This pilot study aimed at verifying the responsiveness of the indicator for different urban patterns. Each set of urban models isolates one important aspect that the indicator should embrace. The results for the proposed urban sprawl indicator, showed in Figure 2, were consistent with what was expected from both visual and statistical analysis (ANOVA).

The first set of examples describes different street network configuration, with different connectivity degrees. We can see that the urban configuration of street network really interferes in the result, since it presents significant difference in the results. The exception is when we compare 1c and 1d (p=0,996) and 1d and 1e (p=0,255), which are actually very similar to one another. We can also notice that the most fragmented example (1e) presented the higher sprawl indicator, while the most connected (1a) presented the lowest value. In this set, since origin and destination are not weighted, distances were computed from each node to all others.

The second set describes different patterns of employment location, ranging from monocentric to polycentric patterns. Again we can see significant differences (p=0,000) in the results, with some few exceptions. More dispersed patterns of employment location resulted in higher scores for the urban sprawl indicator.

Finally, in the third set, we actually have two subsets that describe different distribution of residences, comprising different households and jobs mismatching patterns. Statistical testing (Factorial ANOVA) showed that both factors – jobs location and housing location – that interfere in the result are independent. Then we could realise that, regardless the pattern of residence location, if workplaces are concentrated in the geographic centre of the system the values for the indicator tend to be lower. Likewise, if the residences are located closer to the geographic centre the values tend to be lower because this way distances are shortened. Another interesting finding is that when the workplaces are completely mismatched from residences, it does not matter if the residences are concentrated, like in 3b and 3e, or if they are split, like 3c and 3f, because there is no significant distance between 3b and 3c and between 3e and 3f (p=0,812).

Since the results of the pilot study were satisfactory and showed to be trustful to capture different urban configuration characteristics, next section presents an attempt to establish a method to use the proposed indicator with empirical data.

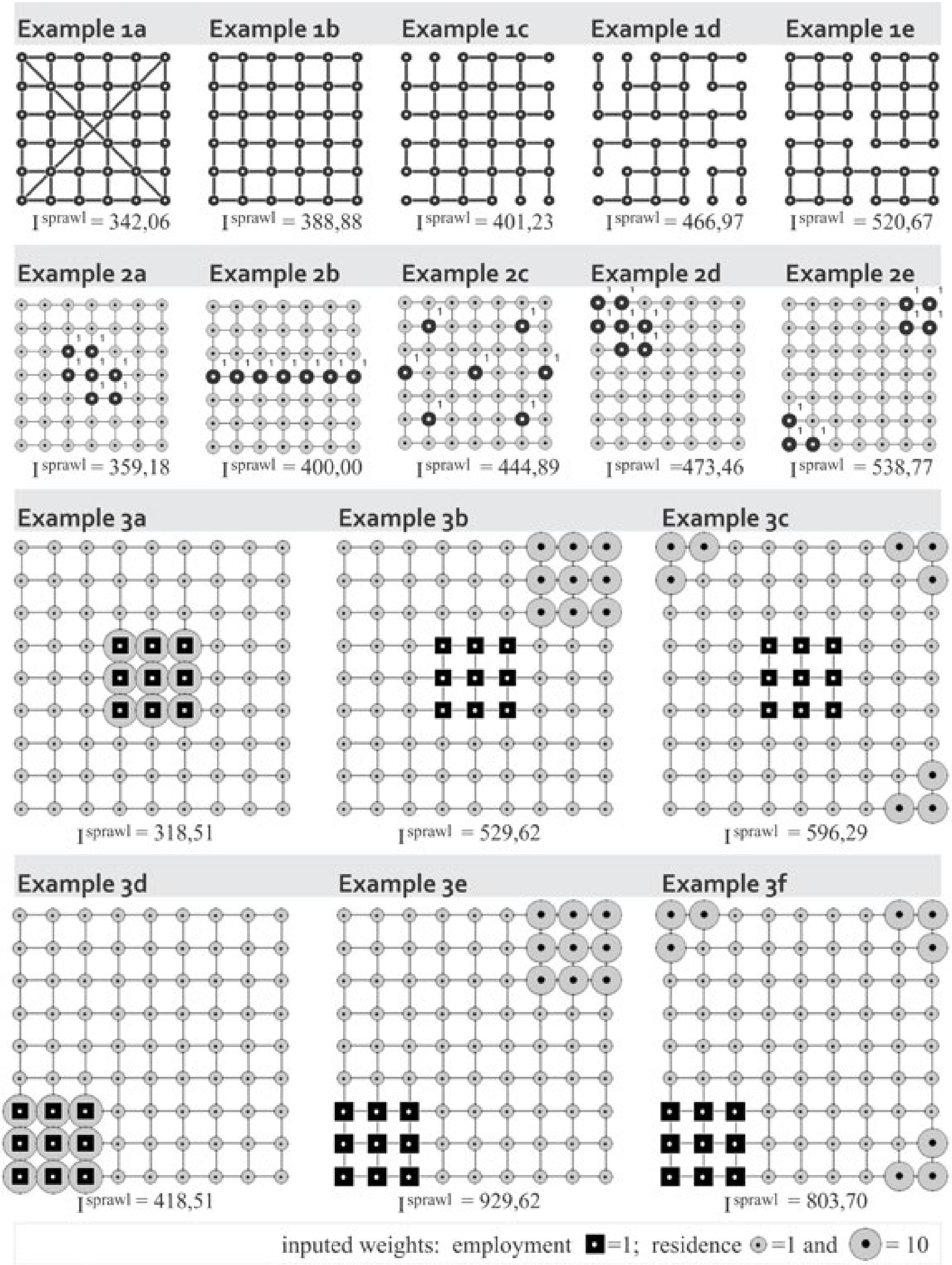

Figure 2 - Pilot study. Different examples of urban configurations and their respective results for the urban sprawl indicator. It is important to highlight that higher values represent the most sprawled examples.

5. CASE STUDY

The study area was the municipality of Torres, in the south of Brazil. It was chosen because of the availability of data. Since 96% of the population is urban, the rural part was excluded from this study. Torres presents one main centre and a few sub-centres. Most of them are near the most important roads, which have attracted new urban settlements since they were reconstructed.

The first step was to construct the urban graph that describes the urban system of Torres. The nodes correspond to the street junctions, however, we assume that they describe small portions of the city containing a sample of urban built forms, which can have attributes about its number of households, its number of workplaces, or both. The lines describe the street network that connects the urban built forms.

Nodes were loaded with attributes of households and workplaces, which can also be read as origin and destination points, respectively. The number of households was obtained from Census 2010. Since we did not have the exact location of workplaces in Torres, we used the commercial places obtained from a mapping done by the city hall. Commercial places were used as a proxy for the number of workplaces, although it is surely a simplification of all possible workplaces in the city. Not all the nodes were loaded with workplaces, but all of them have households. Figure 3 shows the overall distribution of activities loaded in the graph.

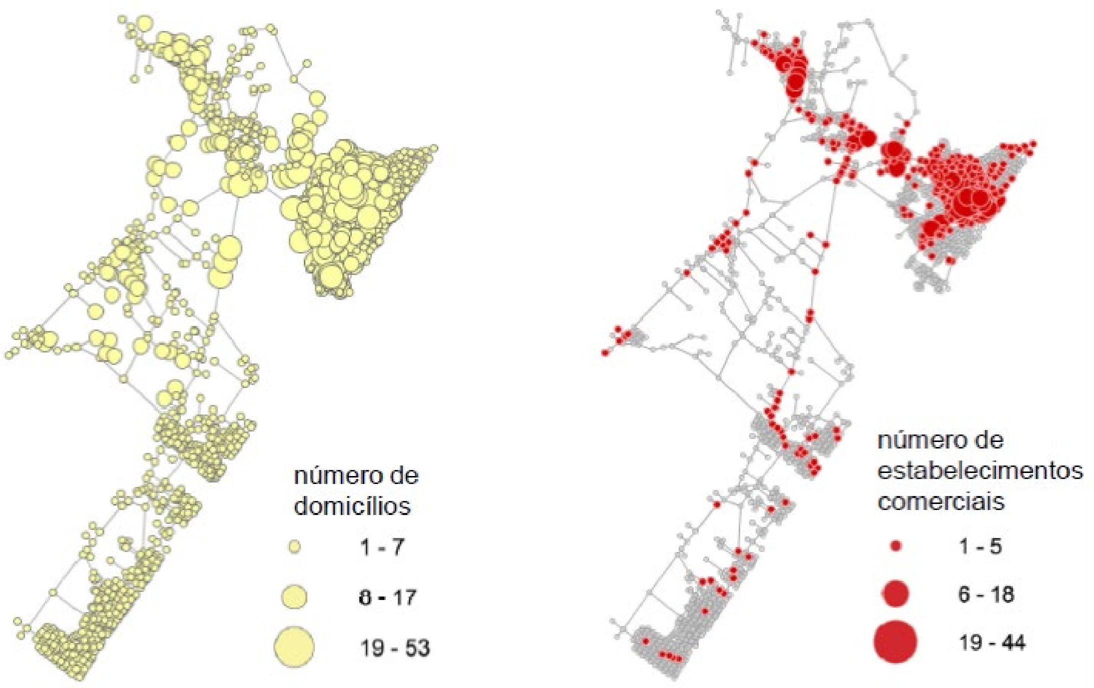

Figure 3 - Input based on empirical data

Since historical data series from Torres were not available, we adopted some hypothetical scenarios for households and workplaces distribution. Therefore, three different spatial patterns of workplaces distribution were used, as well as two different spatial patterns of household distribution.

Figure 4 presents the scenarios used for commerce distribution. Scenario 1 describes empirical data, which resembles the real distribution pattern of commerce facilities. Scenarios 2 and 3 are hypothetical and describe a monocentric pattern and a more dispersed spatial distribution pattern of facilities, respectively.

Scenario 1 Scenario 2 Scenario 3

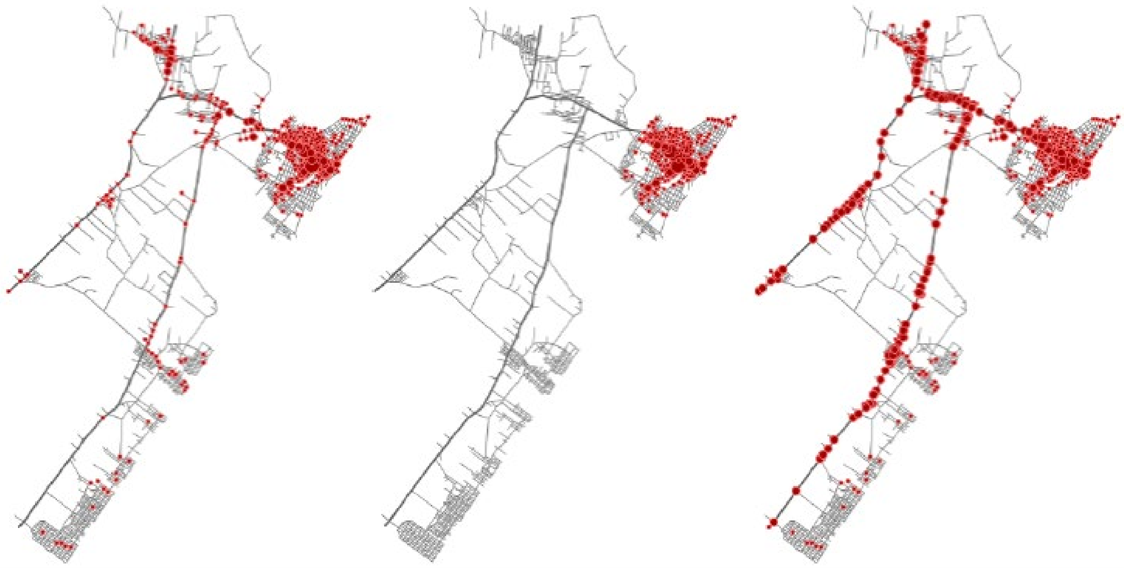

Figure 4 - Spatial distribution pattern of workplaces.

Figure 5 shows the scenarios used for housing distribution. Scenario A describes empirical data, while Scenario B describes a simulation of a more dispersed pattern, in which denser sub-centres were settled far from downtown.

Scenario 1 Scenario 1

Figure 5 - Spatial distribution pattern of households.

Those patterns were combined into six different scenarios of activities distribution and Table 1 shows the results for the urban sprawl indicator (I^{sprawl})

Scenarios	Distribution of households	Distribution of workplaces	I^{sprawl}
A1	Scenario A	Scenario 1	3.869
A2	Scenario A	Scenario 2	3.181
A3	Scenario A	Scenario 3	5.331
B1	Scenario B	Scenario 1	6.144
B2	Scenario B	Scenario 2	5.850
B3	Scenario B	Scenario 3	6.810

Table1 - Final ranking for urban sprawl indicator.

Before discussing the results it is important to highlight that three levels of analysis can be conducted with this model. The first one is calculating accessibility of each node, without any weighting system. The second one is calculating the accessibility of each residence place to every workplace. The result can be read as a local indicator that shows how far each point is from workplaces. This way, only nodes with destination activities – workplaces – are weighted. The third one is the urban sprawl indicator, an aggregated indicator for the whole urban system. Here, origin and destination points are weighted. Therefore, the indicator shows the spatial mismatch between residences and jobs.

The variables that interfere in the result – distribution of household and workplaces – are independent. The statistical analysis (Factorial ANOVA) reveals that there is a significant difference between the results. It also showed that both aspects (distribution of households and workplaces) are independent. The post-hoc test shows that the most dispersed pattern of workplaces (Scenario 3) tends to increase distances, regardless the housing distribution pattern of households. The same occurs with households.

The results also showed that the more concentrated the workplaces are in highly accessible places, the less sprawled the system is. This finding should be more deeply investigated since it seems to reveal that decentralising jobs may not be a powerful urban policy.

6. FINAL CONSIDERATIONS

Previous studies about urban sprawl measurement have explored mainly indicators based on density and general urban form. Nevertheless, none of the measures developed up to now has focused on accessibility and intra-urban details. Aiming to fill this gap, this study presented an alternative accessibility-based indicator. The main advantage over the current methodologies proposed in sprawl studies is its stronger relationship with sprawl impacts. As already claimed, density and urban form's measures have limited power to direct urban policies. We need to improve methods for intra-urban level analysis to tackle urban performance issues.

Besides verifying accessibility to employment opportunities and spatial mismatch between urban activities, this study demonstrated that the proposed indicator could also capture how far activities are from the most accessible places – considering here accessibility from each place with all other ones.

Empirical research should be extended to reveal the capability of the indicator, based on weighted accessibility measures as a decision-support tool. The indicator could be compared with real traffic, pollution and fuel consumption data to produce real urban performance indicators.

Further work could also join the accessibility-based indicator with other types of sprawl measurement indicators to improve a multidimensional approach. Outside the sprawl measurement context, the methodology could also be applied to specific origin and destination input, such as poor household location, employment opportunities, and more.

REFERENCES

Batty, M. (2009), "Notes on Accessibility: In Search for a Unified Theory'. I: Koch, D., Marcus L. and Steen J. (eds), *Proceedings of the Seventh International Space Syntax Symposium*, Stockholm: Royal Institute of Technology, p.102.1-102.4.

Bertuglia, C., Clarke, G. and Wilson, A. (1994), *Modelling the City: performance, policy and planning*, London: Routledge.

Bertaud, A. and Malpezzi, S. (2003), 'The Spatial Distribution of Population in 48 World Cities: Implications for Economies in Transition', in *Report*. Centre for Urban Land Economics Research, University of Wisconsin.

Bertaud, A. and Malpezzi, S. (2014), 'The Spatial Distribution of Population in 57 World Cities: The Role of Markets, Planning and Topography', in Report. Centre for Urban Land Economics Research, University of Wisconsin.

Chin, N. (2002), 'Unearthing the Roots of Urban Sprawl: A Critical Analysis of Form, Function and Methodology', in CASA Working Paper 47. London: Centre for Advanced Spatial Analysis, University College London.

Ewing R., Pendall, R. and Chen, D. (2002), 'Measuring Sprawl and its Impact', in *Smart Growth America Report*., Vol. 1.

Ewing, R. and Hamidi, S. (2014), 'Measuring Urban Sprawl and Validating Sprawl Measures', in *Smart Growth America Report*.

Franz, G., Maier, G. and Schröck, P. (2006), 'Urban Sprawl: How Useful is this Concept? In: *Proceedings of 46th Congress of the European Regional Science Association (ERSA)*. Paper 6, p. 1-29.

Frenkel, A. and Ashkenazi, M. (2008), 'Measuring urban sprawl: how can we deal with it', in *Environment and Planning B: Planning and Design*, Vol.35, p.56-79.

Galster, G., Hanson R., Ratcliffe M., Wolman H., Coleman S. and Freihage J. (2001), 'Wrestling Sprawl to the Ground: Defining and Measuring an Elusive Concept', in *Housing Policy Debate*, Vol.12, p.681-717.

Haynes, K. and Fotheringham, A. (1984), *Gravity and spatial interaction models*, Beverly Hills: Sage.

Hillier, B. and Hanson, J. (1984), *The Social Logic of Space*, Cambridge: Cambridge University Press.

Ingram, R. (1971), 'The concept of accessibility: a search for an operational form', in *Regional Studies*, Vol. 5, p. 101-107.

Jaret, C., Grade, R., Reid, L. and Adelman, R. (2009), 'The measurement of suburban sprawl: an evaluation', in *City & Community*, Vol.8 (1), p.65-84.

Krafta, R. (1994), 'Modelling intraurban configurational development', in *Environment and Planning B: Planning and Design*, Vol.21, p.67-82.

Krüger, M. (1990), 'On node and axial grid maps: distance measures and related topics', in CASA Working Paper 30. London: Centre for Advanced Spatial Analysis, University College London.

Longley, P., Batty, M. and Chin, N. (2002), 'Sprawling cities and transport: preliminary findings from Bristol, UK', in *ERSA conference papers*, European Regional Science Association, Centre for Advanced Spatial Analysis and Department of Geography, University College, London.

Netto, V. and Krafta, R. (2009), 'A forma urbana como problema de desempenho: o impacto de propriedades espaciais sobre o comportamento urbano', in *Revista Brasileira de Estudos Urbanos e Regionais*, Vol. 11, p. 157-182.

Ribeiro, R. and Holanda, F. (2006), 'Proposta para Análise do Índice de Dispersão Urbana', in *Cadernos Metrópole, Observatório das Metrópoles*, Vol. 15, p. 49-70.

Song, S. (1994), 'Modelling Worker Residence Distribution in the Los Angeles Region', in *Urban Studies*, Vol. 31(9), pp. 1533-44.

Torrens, P. (2008), 'A toolkit for measuring sprawl', in *Applied Spatial Analysis and Policy*, Vol.1, p.5-36.

Torrens, P. and Alberti, M. (2000), 'Measuring Sprawl', in CASA Working Paper 27. London: Centre for Advanced Spatial Analysis, University College London.

Wasserman, S. and Faust, K. (1994), *Social network analysis: methods and applications*, Cambridge: Cambridge University Press.

Wilson, A. (1971, 'A family of spatial interaction models and associated development', in *Environment and Planning*, Vol.3, p. 1-32.

Wilson, A. (1974) *Urban and regional models in geography and planning*, London: J. Willey.