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ISOVIST BASED ANALYSIS OF SUPERMARKET LAYOUT

Verification of Visibility Graph Analysis and Multi-Agent Simulation

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ABSTRACT

Supermarket floor layout has influence on customers shopping behaviour and movement path. However, past findings are mainly concentrated on layout patterns identification, lacking computational evaluation of real world layouts. In former research, algorithms in Depthmap such as VGA (Visibility Graph Analysis) and MAS (Multi-Agent Simulation), based on the core concept of isovist, are proved effective in quantitative evaluation of spatial traversability and attractiveness. These algorithms could be new approaches for evaluating supermarket floor layouts.

This paper is intended to verify the application of VGA and MAS in supermarket layout analysis by comparing their calculation results with both sales experiences and real world data. Firstly, typical layout patterns are analysed with the two algorithms to verify their consistencies with sales experiences. Secondly, calculations of an experiment supermarket are conducted with parameters adjusted according to the actual environment; tracking points of customers are also collected with a high-precision indoor tracking system. Correlations between the calculation results and real world collected data are demonstrated.

The main conclusion is that VGA has its value in position evaluation however cannot predict people flow, while MAS result can be an important reference to both layout evaluation and customer flow prediction. Additionally, applications and some possible refinements of the existing algorithms are also suggested for future studies.

KEYWORDS

Isovist, Visibility Graph Analysis, Multi-Agent Simulation, Supermarket, Indoor Tracking, Layout Analysis

1. INTRODUCTION

In the research realm of interior retail environment, floor layout design is vital as it exerts spatial influence on in-store traffic patterns, shopping atmosphere, shopping behaviour and operational efficiency (Lewison, 1994), which further significantly impact commercial performances such as price acceptability, shopping quantity, store loyalty and etc. , (Merrilees & Miller, 2001; Levy & Weitz, 2001; Underhill, 2000). In the first stage of layout studies, most findings focus on the form of guidelines on technological constraints, such as minimizing the distance between refrigeration system and electrical power sockets (Boros et al, 2016). With the advance of equipment and prosperity of marketing theories, the focus shifted to layout patterns and category allocation. The summarized typology of layout patterns - gird, race track, free form and circulation spine, are applied and analysed (Levy & Weitz, 2001; Lewison, 1994) in complementary with marketing and customer studies. Meanwhile, types of product assortment are investigated (Borges, 2003; Cil, 2012) and experiments on shelf orientations adjustment are conducted (Hwang & Lee, 2005). On the other hand, quantitative techniques, analytical methods and simulation models has emerged to evaluate a given floor layout, including customer flow simulation applying the concept of Traveling Salesman Problem (Boros et al, 2016; Misevicius, 2005; Aleisa & Lin, 2005). In a word, during recent decades, the study of supermarkets layout has developed to be more quantitative and involved with customer research owing to advanced algorithms.

Since Bill Hillier suggested that spatial form predetermines the function of space and human activities (Hillier, 1996), people began to conduct objective analyses of space to tackle practical problems. Based on isovist concept and Space Syntax theory, a collection of algorithms and tools including Axial Map Analysis, Convex Map Analysis, Visibility Graph Analysis, Segment Map Analysis and Multi-Agent Simulation have been developed and applied to a wide range of scales of spaces and compatible occasions (Al-Sayed et al, 2014). These analysis tools exclude social, functional and other factors to reveal the impacts that space imposes on man to pursue objective and quantitative outcomes. Among these tools, VGA (Visibility Graph Analysis), Convex Map Analysis and MAS (Multi-Agent Simulation) are more widely used in building scale. Unlike Convex Map Analysis that only fits the type of building with explicitly defined rooms, VGA and MAS have a much wider application.

VGA and MAS have been carried out in urban context or art galleries analysis and have proved high consistent with distribution of people in real situations (Duan & Hillier, 2015). The compatibilities of these tools for supermarket scene still remains to be examined. Unlike other approaches, such as TSP (Travelling Salesman Problem) algorithm which tackles the analysis situation as a mathematic model, VGA and MAS aim to solve the problem from a cognitive perspective.

To verify the applicability of VGA and MAS in supermarket layout analysis, this paper is organized as follows. In the first part, this paper introduces the concept of isovist and explains the principles of VGA and MAS, especially their relationships with isovist. In the second part, some typical layouts are analysed with VGA and MAS to demonstrate the relevance between analysis results and classic sales experiences. In the third part, the effectiveness of the methods is verified quantitatively with position evaluation experiences and real world data collected with a precise indoor positioning system in an experiment supermarket. The position value indicator (Magnetic related/All) and R² (coefficient of determination) are calculated to quantify the consistency. At last, this paper discusses the limitation of tracking technology, and proposes further algorithm refinements.

2. DATASETS AND METHODS

2.1 ISOVIST, VGA AND MAS PRINCIPLES

After the idea of isovist/isovist field was proposed by Benedikt, it offers a new insight into how people navigate through space. An isovist is the set of all points visible from a given vantage point in space and with respect to an environment (Figure1), and it forms an alternative

description of environments to further understand view control, privacy and defensibility in dynamic complexity and spaciousness judgments (Benedikt, 1979). Benedikt supposed that people would be guided by isovist properties rather than objects, while Gibson further suggests that people may be guided by direct (or active) perception without higher cognitive functions (Gibson, 1979). As it is agreed that vision is the most critical sense upon which most humans are dependent in environment sensing and reacting, some algorithms are developed based on the isovist concept to predict people movements. VGA and MAS are just two typical algorithms representing modelling and simulation approaches respectively to address the issue of pedestrian movement analysis (Penn & Turner, 2001).

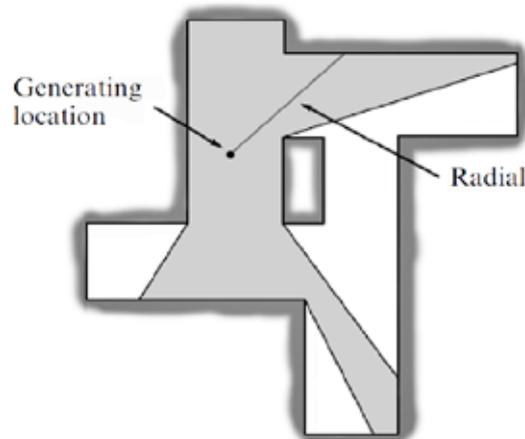


Figure 1 - An isovist polygon representing the visible area from a generating location (Benedikt, 1979)

VGA is a spatial analysis method combining isovist concept with space syntax theory, which can be shown on a grided graph with the grids coloured according to their spatial properties. Hillier and Hanson developed the theory of space syntax and created various representations for the components of space; they then drew maps of these components, and crucially the relationships of the components with each other (Turner, 2004). The properties are statistics of isovist characteristics and visibility configurational relationships. Isovist characteristics include Isovist Area, Isovist Compactness, Isovist Occlusivity, Isovist max Radial and others. Visibility configurational relationships of grids laid, can be either global or local, i.e., for each vertex and for the entire system, include Visual Control, Visual Controllability, Visual Integration, Visual Clustering Coefficient and others. These properties display differences in cognition of space, in other words, people in different behaviour objectives may take some properties prior to others in their actions. In former researches, different properties are selected to fit in proper scenes (Orellana & Alsayed, 2013; Kalff et al, 2010). VGA outcomes are proved efficient as the past findings show that between 50% and 80% of the variance in pedestrian flows from location to location in an environment can be explained in terms of variations in configurational properties of those locations in the network (Penn & Turner, 2001).

The isovist based method MAS, utilizing vision as the driving force in path planning that developed by Penn and Turner, has been proved to match the movements of real pedestrians by a simple rule (Penn & Turner, 2001). It is proved efficient to predict the collective behaviour emerged from individual random walks constrained by geometry but aided with what agents can see in different scales of scenes (Batty, 1993). The simulation rule of agents can either use VGA properties or just as simple as a random next step rule. In fact, the simplest random next step rule performed better than the complicated rule which uses Visual Clustering Coefficient data in the retailing store experiment (Penn & Turner, 2001). So the simplest random next step rule is recognized as the default rule. This isovist based simulation is different from others in that it doesn't assure a certain destination or assume a rational choice in order to optimize some measure of cost such as travel time.

These two algorithms are both implemented in Depthmap, which is a program developed by Alasdair Turner (Turner, 2003). The Working process is shown in Figure 2. Before applying to different scenes, some parameters need to be decided for MAS; on the other hand, the most relevant properties among VGA should be picked up. In this study, the properties and parameters are decided according to the key points mentioned in layout experiences.

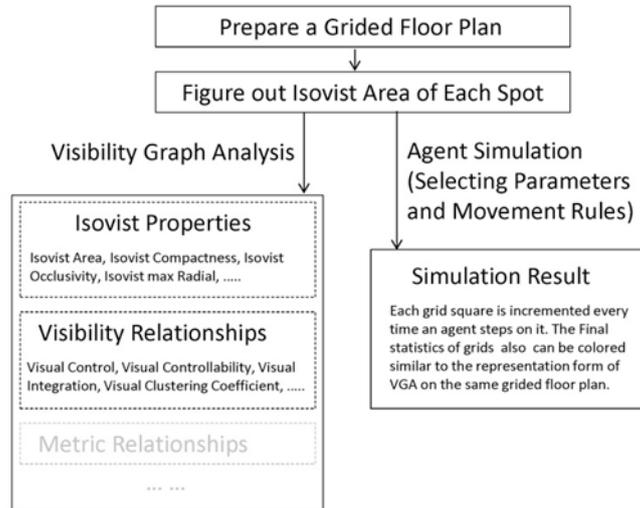


Figure 2 - Depthmap working process of VGA and MAS

2.2 PROPERTIES AND PARAMETERS DECIDING

The first step is to decide the grid size for subdividing the supermarket plan. It is set at 0.6*0.6 meters as it's the average step length and also the passing through width of an adult, which facilitates both MAS with the proper step length and VGA with the proper human scale. Although the program only takes the grid location but not the grid square, this setting makes it easier to relate real human steps to the steps taken in simulation, and achieving a balance between accuracy and computing time.

Conventional retailing store layout theory has summarized three typical types of store layouts, namely Grid, Freeform and Racetrack/boutique, with each one having its advantages and disadvantages (Lewison, 1994; Levy & Weitz, 2001). The grid layout is a rectangular arrangements of displays and long aisles that facilitates routine and planned shopping behaviour, providing flexibility and speed in identifying products; Freeform gives customers enough freedom to move in any direction and increases time spent in the shop; racetrack/boutique leads the customer with well-organized structure and creates shopping entertaining experiences. The most commonly used layout in a supermarket is the grid pattern, based on the assumption that at least half of the customers already have clear motives when they enter the shop.

In the descriptions of how the patterns facilitate customers, some key characteristics can be identified. Routine and shopping time are main concerns of the whole shopping process, while freedom of choice, routine length and structure are also mentioned as important spatial issues. According to these aspects, the four properties of Isovist Area, Isovist Occlusivity, Visual Control and Visual Integration are selected from VGA properties as most related to customer distribution. These four properties are defined as follows:

Isovist Area: Isovist Area counts the number of nodes that are visible from a certain location. As the visible defined in the program is undirected, the spot with higher Isovist Area in a plan has a large vision and also has high visibility from other parts of the space.

Isovist Occlusivity: Occlusivity accounts for parts of the isovist area that are occluded but permeable. It implies the degree of closure of a certain area.

Visual Control: Control picks out visually dominant areas - in order to be controlling, a point must see a large number of spaces, but these spaces should each see relatively little (Turner, 2004). Areas with high control value always play an important role in the spatial structure.

Visual Integration: Integration is an essentially normalized version of the mean depth. Former researches have proved its correlation with pedestrian flow amounts - the higher integration, the more possible to occur large amounts of pedestrian flow (Hillier et al, 1993).

There are four parameters in MAS that would affect agent movements: the number of steps before turn decision, field of view, timesteps in system, and system analysis length. The default value is 15bins of field of view equivalent to 170° and 3 steps before turn decision. The default value is proved correlating well to real situations in former research (Turner and Penn, 2002). However, in the experiment conducted in an airport, the author adjusted the step number to 6 to make the simulation path better fit individual navigation path, resulted in a better correlation rate (Orellana & Alsayed, 2013). As the common width of supermarket aisle is about 1.2 meter, plus the 0.9 meter width of shelf, 2.1meter will be the distance between two adjacent aisles. In this sense, the 2.1 meter length has a span of 3.5 grids with 0.6 meters sides, assuming that the 3 steps turn parameter is reasonable in the supermarket scene. Timesteps in system in the experiments are decided assuming the customers go through all the aisles in the supermarket, so it can be calculated according to the grid numbers of the plan. System analysis length is four times of agents timesteps to ensure a relatively stable status.

3. THEORETICAL ANALYSIS BASED ON SALES EXPERIENCES

3.1 FIGURING OUT MAGNETIC POINTS

One type of sales experience analysis of a supermarket layout is to identify the most valuable positions in a given plan. Those positions are called Magnetic Points (Katsutoshi Niiyama & Hu, 2011), which usually appears at the end of shelves along the main aisle, or in the open area in front of the cashier. Items around these points have to be carefully arranged in order to attract and guide customers to make them stay longer in the shop.

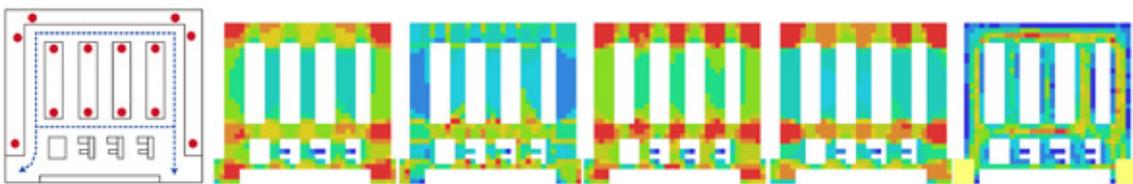


Figure 3 - Magnetic Points in a supermarket, VGA and MAS results (From left: isovist area, isovist occlusivity, visual control, visual integration, MAS)

To testify validities of VGA and MAS, the two methods are applied on the given example showing magnetic points (Katsutoshi Niiyama & Hu, 2011), and the results are shown in Figure3. The parameters of MAS are set 15 bins of field of view and 3 steps before turn decision. The timesteps is set to 200 (total grid number 768) and the system analysis length is set to 800. The agents are released at the entrance's location.

Seen from the VGA graphs, the Magnetic Points are located near the high isovist, high visual control and high visual integration locations. In other words, these three graphs reveal the locations that have the highest privilege to be seen and surpassing by customers. Seen from the MAS graph, the main aisle drawn in the experience layout is passed by most agents, which directly represent people flow.

3.2 REPRESENTING LAYOUT RULES

Another type of supermarket layout experiences is the layout rules- dos and don'ts about how to organize the shelves. Figure 4 shows two inefficient layouts that couldn't lead the customers go into vast areas of a supermarket. From the analysis results, we can see only the MAS shows some similarities with the experience, while none of the VGA results reflects the experience. This reveals the differences between the two approaches that the VGA algorithm may not successfully represent real customer movement. When switch to a maze plan as in Figure 5, the differences can be seen more clearly. The agents formed a path through the maze which may occur in real situations, while the VGA can't directly represent movement. From here, the assumption is raised that even VGA approaches have been proven to correlate with pedestrian flow in urban spaces and indoor space, such as galleries, museums, and large commercial buildings, they may not be fit for supermarket environment with narrow passages and explicit entrances. On the other hand, the MAS is able to represent the possibilities of customer flow to a certain extent.

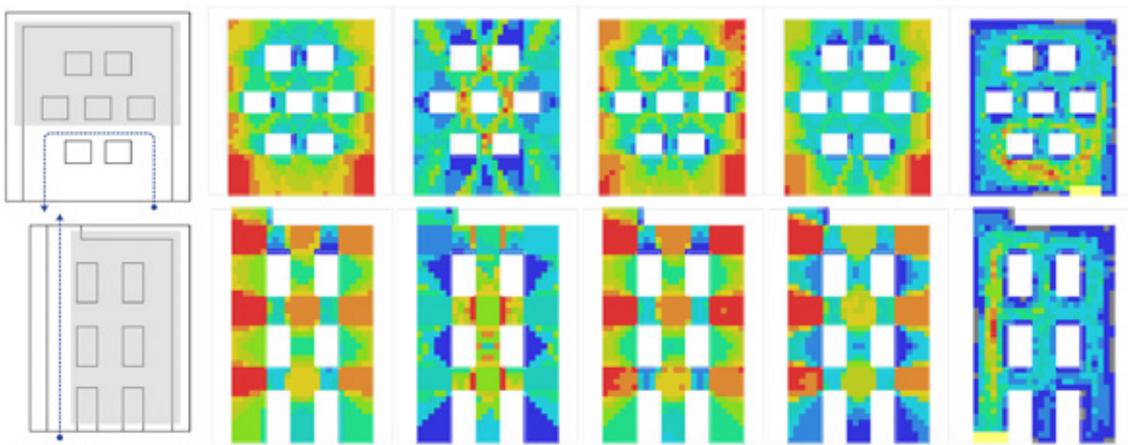


Figure 4 - Two inefficient layouts, VGA and MAS outcomes (from left: isovist area, isovist occlusivity, visual control, visual integration, MAS)

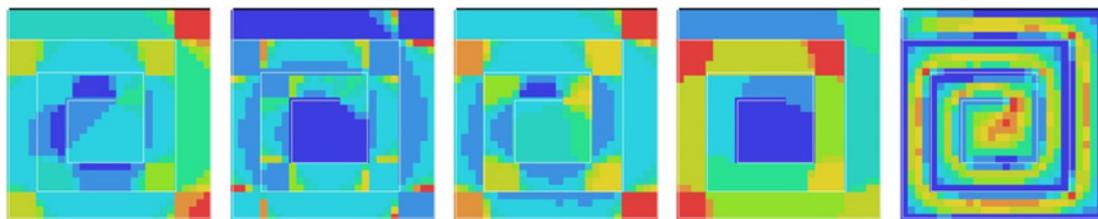


Figure 5 - The VGA and MAS result of a maze (from left: isovist area, isovist occlusivity, visual control, visual integration, MAS)

Concluded from these two theoretical analyses, a preliminary conclusion can be drawn that VGA has its value in position evaluation but cannot predict people flow, while MAS result can contribute to layout evaluation and prediction of customer flow; and among the VGA results, visual control property is the most effective one. However, to what extent is the VGA result correlated with real people flow caused by spatial effects and what kind of errors may occur are to be discovered quantitatively by real world experiments.

4. QUANTITATIVE VERIFICATION WITH REAL WORLD DATA

An experiment is conducted in a supermarket with a medium size of 40*25 meters, arranged in a typical grid layout. Entrances and cashiers are marked on the floor plan in Figure 6. Because the behaviour of vegetable and fruit purchasing includes cues and weigh, it is excluded from

the studied area. Also, the cashier and exit region at the bottom right and the top left corner of equipment charging area are excluded due to the inevitable data deviation. The merchandise is roughly divided into five groups, including dairy product and drinks, daily necessities and snacks, wine and beverage and frozen food. The arrangement is relatively balanced for household shopping, eliminating the effects of people’s preferences for different merchandise to a great extent. Depthmap calculated outcome of isovist area, isovist occlusivity, visual control, visual integration, visual integration R₃ and MAS are shown in Figure7 with a unified colour legend of red to blue representing high to low value of statistics. The parameter of MAS is set 15 bins of field of view equivalent to 170°, 3 steps before turn decision, 325 timesteps and 1400 system analysis length.

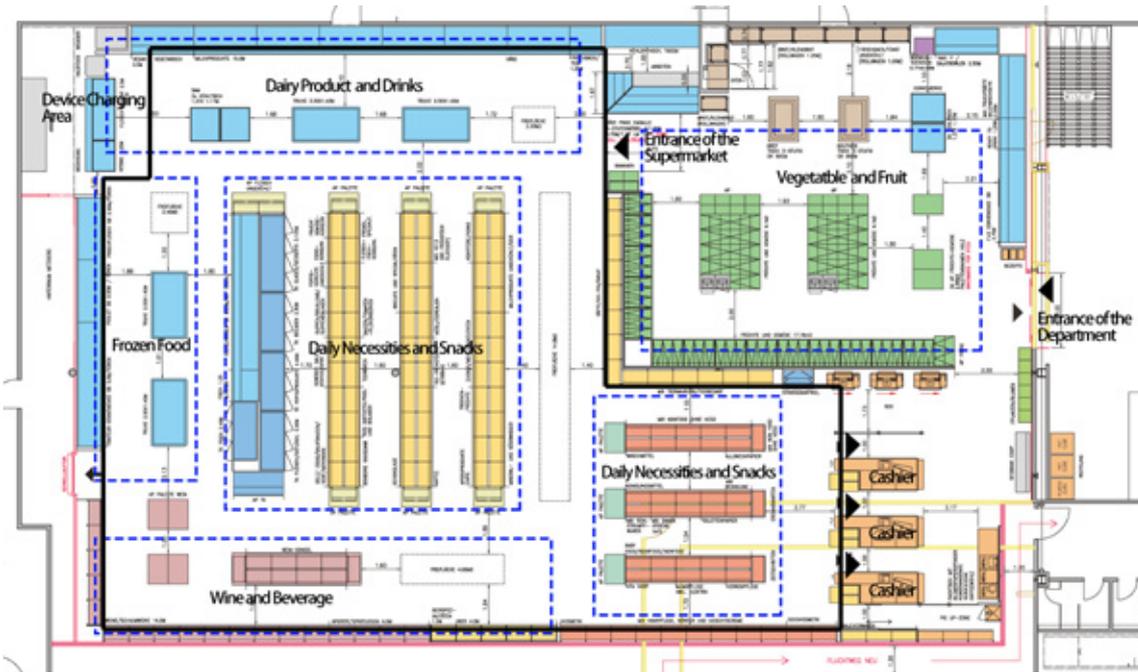


Figure 6 - The floor plan and merchandise arrangement of the supermarket

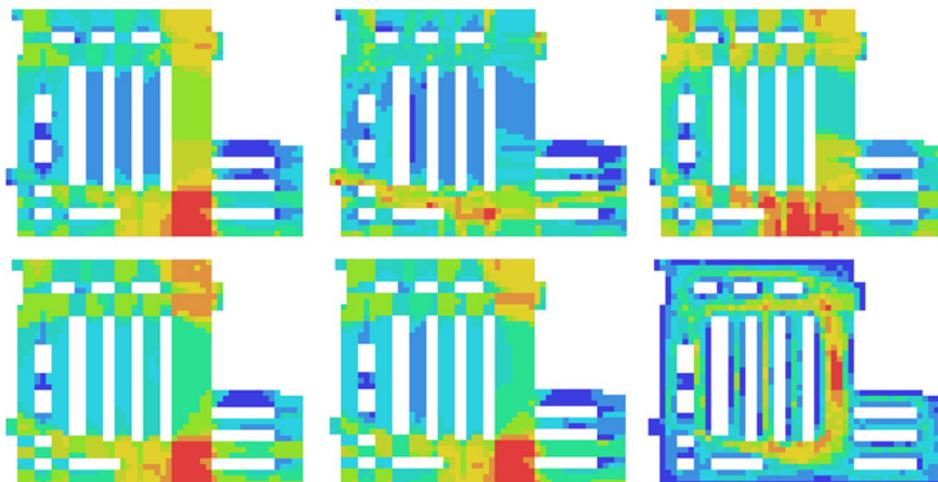


Figure 7 - Depthmap calculation results (from left: isovist area, isovist occlusivity, visual control, visual integration, visual integration R₃, MAS)

The technical problem is how to record and describe people flow. In former studies, the most commonly used method is person following (Batty, 1993). However, this method is not feasible in a supermarket because of the small scale of space and psychological interference on customers. Gate count method is also not feasible in such small scale. In this experiment, we use the distribution of tracking points to represent the people flow. It serves as a more precise method which can overcome the drawbacks of other methods. Positions of customers inside the supermarket are recorded by a high-precision indoor tracking system based on Ultra-wide Band. Tracking devices called "tags" are attached on the shopping baskets. As the participants are asked to take a basket with them at the entrance, the position of the customer is represented by the position of the basket. The sampling interval is 10 seconds. After the experiment, tracking points are overlapped in the plan to form a location point graph. Although the distribution of tracking points dose not exactly equal to people density, by reducing the sampling interval or enlarge the recording period, it can approach the actual people flow infinitely. The tracking system recorded 639 tracking points in the process, and they are shown in Figure8. To compare the recorded data and calculation results, the plan is divided into 17 parts according to the main aisles. There are altogether 1257 grids within the analysed area.

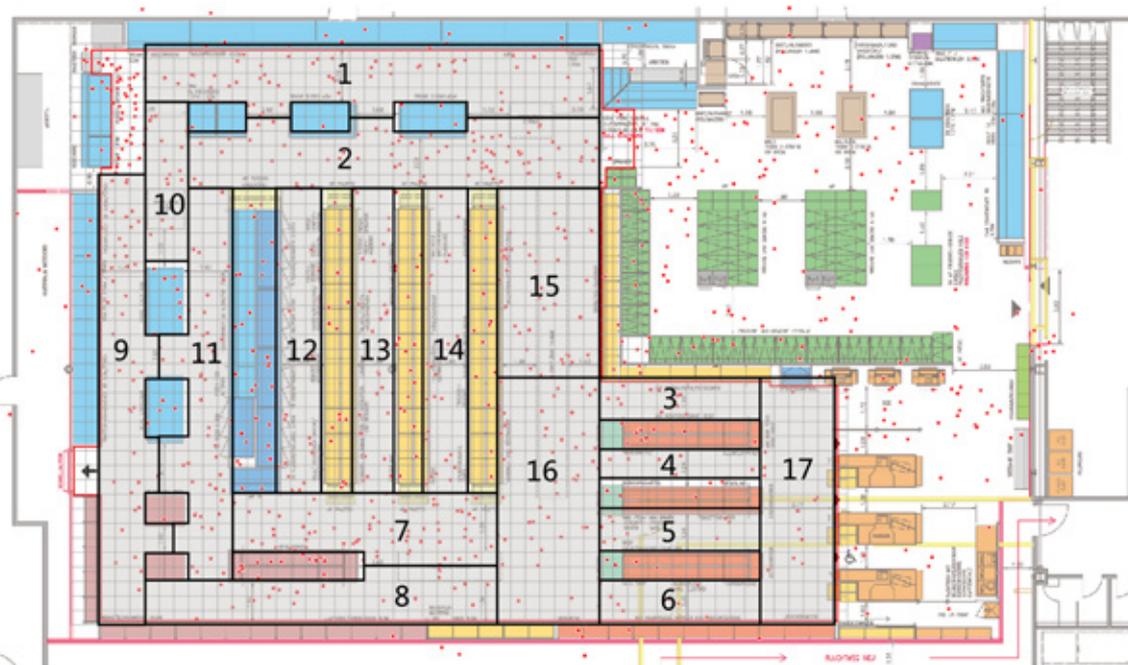


Figure 8 - Tracking points and region index of the supermarket plan

4.1 VERIFICATION OF POSITION EVALUATION

According to the alliance of shelves, the magnetic points and main isles in the supermarket are marked as in Figure9. Regions highlighted represent areas that magnetic points have the strongest influence on. As the sales experience suggests, these regions have higher commercial values, which in other words should enjoy more possibilities to be seen by customers and have more people passing by.

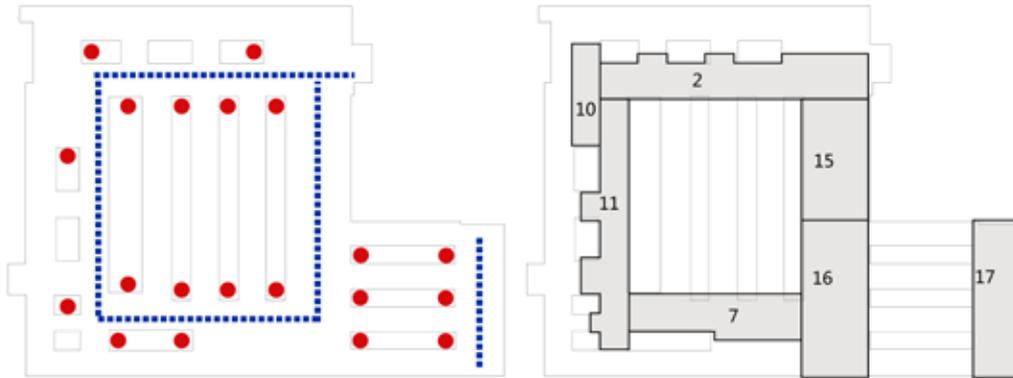


Figure 9 - Magnetic points and corresponding regions of the supermarket plan

The statistics of calculation results are listed in Chart1. The average statistics of highlighted regions, called 'magnetic points related regions', are compared with the whole supermarket's average level, making the 'position value indicator' (magnetic related/ all). The highest ratio is 1.27 in the gate count statistic volume, while the isovist area and visual control property are the second and third highest ratio of 1.18 and 1.10. It can be seen from the chart that the grids in the highlighted regions have a higher level in both VGA and MAS results, proving that these methods both works to identify the most commercially valuable positions in the supermarket plan. In this sense, the applications of these two methods are verified.

Index	Grid Counts	Isovist Area	Isovist Occlusivity	Visual Control	IntegrationH H	IntegrationR 3	Gate Count
1	139	96.6795	34.3542	1.0924	8.8482	8.8829	27.0144
2	127	103.3890	37.9857	1.1354	9.0938	9.1226	48.4409
3	33	34.9617	16.7064	0.6273	5.0737	5.4893	22.5152
4	22	46.4888	30.4568	0.5944	6.2857	6.3594	20.8182
5	33	81.9660	52.1800	0.8155	8.6058	8.5058	23.000
6	33	86.6096	36.6220	0.8533	8.5678	8.5678	21.7879
7	75	115.6580	64.1327	1.3166	9.8173	9.8185	53.5733
8	81	109.7650	48.5161	1.1934	9.4104	9.4107	19.5679
9	104	70.7813	34.2300	0.9264	7.3670	7.4750	18.9712
10	33	81.9078	41.9768	1.0566	7.5089	7.6374	38.2727
11	97	63.6659	34.5320	0.8805	6.8901	7.0241	29.4536
12	63	39.7203	19.6703	0.7669	6.1102	6.1795	23.6032
13	63	47.9780	19.4437	0.7117	6.5800	6.5930	35.4762
14	63	45.7434	23.0850	0.6759	6.7910	6.8046	26.9206
15	91	112.1740	22.6017	0.8729	8.0879	8.1100	51.6923
16	119	154.5180	42.9795	1.3940	10.6399	10.6533	45.4958
17	81	61.8223	27.4881	0.9349	6.7828	6.8433	18.6420
Magnetic Related	623	64286.39	23766.83	684.25	5358.90	5388.39	25917.99
Average	-	103.19	38.15	1.10	8.60	8.65	41.60
All Regions	1257	110123.37	44102.15	1253.69	10216.96	10283.92	41328.00
Average	-	87.61	35.09	1.00	8.13	8.18	32.88
Manetic Related/All		1.18	1.09	1.10	1.06	1.06	1.27

Table 1 - The calculation statistics and position value indicator (magnetic related regions/all)

4.2 VERIFICATION OF PREDICTING PEOPLE FLOW

Density of tracking points in each region is calculated for comparison with VGA and MAS outcomes. The correlation is represented by R2 (coefficient of determination) between tracking points density and Depthmap results, as listed in Chart2. From the chart, we can see that the R2 of tracking points density and MAS is 0.6544, indicating a strong correlation. In the correlation chart, it shows that these two points that most deviated from the trend line are index 15 and 16, with fewer tracking points than expected. Generally, the MAS method can effectively predict customer distribution in a supermarket. On the other hand, the R2 of tracking points density and VGA is much lower, with the highest value of 0.1192 of visual control, indicating a weak correlation. So it can be concluded that all of the VGA analyses failed to predict customers' behaviours in a supermarket.

This result is almost consistent with the preliminary conclusion drawn from the theoretical verification that VGA has its value in position evaluation but cannot predict people flow, while MAS result can contribute to layout evaluation and customer flow prediction.

Index	Grid Counts	Recorded Location Points	People Density (per grid)	Isovist Area	Isovist Occlusivity	Visual Control	IntegrationH H	IntegrationR 3	Gate Count
1	139	66	0.4748	96.6795	34.3542	1.0924	8.8482	8.8829	27.0144
2	127	91	0.7165	103.3890	37.9857	1.1354	9.0938	9.1226	48.4409
3	33	15	0.4545	34.9617	16.7064	0.6273	5.0737	5.4893	22.5152
4	22	10	0.4545	46.4888	30.4568	0.5944	6.2857	6.3594	20.8182
5	33	10	0.3030	81.9660	52.1800	0.8155	8.6058	8.5058	23.000
6	33	12	0.3636	86.6096	36.6220	0.8533	8.5678	8.5678	21.7879
7	75	76	1.0133	115.6580	64.1327	1.3166	9.8173	9.8185	53.5733
8	81	21	0.2593	109.7650	48.5161	1.1934	9.4104	9.4107	19.5679
9	104	43	0.4135	70.7813	34.2300	0.9264	7.3670	7.4750	18.9712
10	33	19	0.5758	81.9078	41.9768	1.0566	7.5089	7.6374	38.2727
11	97	53	0.5464	63.6659	34.5320	0.8805	6.8901	7.0241	29.4536
12	63	28	0.4444	39.7203	19.6703	0.7669	6.1102	6.1795	23.6032
13	63	40	0.6349	47.9780	19.4437	0.7117	6.5800	6.5930	35.4762
14	63	29	0.4603	45.7434	23.0850	0.6759	6.7910	6.8046	26.9206
15	91	49	0.5385	112.1740	22.6017	0.8729	8.0879	8.1100	51.6923
16	119	64	0.5378	154.5180	42.9795	1.3940	10.6399	10.6533	45.4958
17	81	13	0.1605	61.8223	27.4881	0.9349	6.7828	6.8433	18.6420
R² Results with People Densiy (per grid)				0.0758	0.0919	0.1192	0.0724	0.0759	0.6544

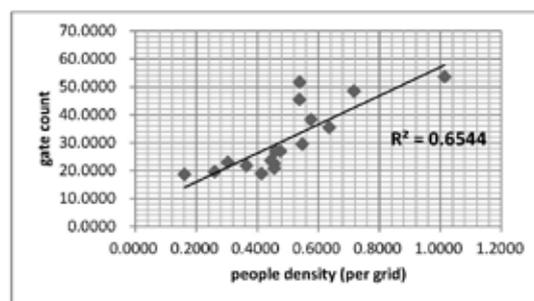
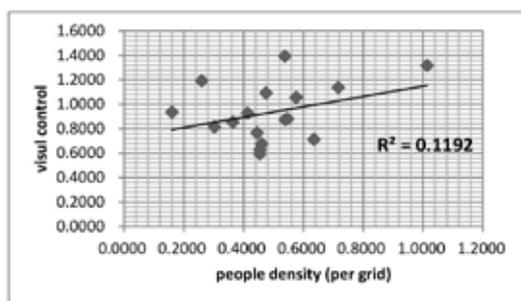


Table 2 - Recorded people density, Depthmap results and their correlations

5. LIMITATIONS AND CONCLUSIONS

Due to the limitation of the tracking system, the sampling frequency is not high enough to record the actual moving track of a certain customer. So the distribution of tracking points is used in the comparison instead of the real track, which could lead to some deviation in the result. Future research work may explore some improvements of algorithms. The MAS now used in Depthmap can work out a relatively satisfying result to correlate with real situation; however, the parameters can be adjusted to fit the supermarket scene better. First of all, the timesteps value that removes the agent after a certain running period should be replaced by a group of various values, since customers don't spend the same amount of time in the supermarket.

Floor layout design plays an important role in supermarket design, since it directly affects people traffic pattern and behaviour, thus influences sales volume. This study tries to verify the effectiveness of VGA (Visibility Graph Analysis) and MAS (Multi-Agent Simulation) on the supermarket layout analysis. The two analytical tools each represents modelling and simulating approach, and embodies its own technical algorithm. Meanwhile, they both are based on the same idea that vision is the upmost important human sense to receive environmental information and leads movement, and are developed based on the concept of isovist which defines the area visible from a given point.

By comparing algorithm results, marketing experiences and real world data, the effectiveness of these two methods are evaluated. The conclusion is that VGA has its value in position evaluation but cannot predict people flow, while MAS result can be an important reference to layout evaluation and customer flow prediction. Additionally, some possible refinements of the existing algorithms are also proposed for future studies.

The results are of commercial values. As it is verified to predict customer density caused by spatial effect, the comparison of calculation result and real customer allocation can be used to analyse marketing issues such as packaging attractiveness, advertising, price and other commodity properties. The sales amount is more feasible to analyse separately with positioning factor and other factors. Besides, the study is meaningful in the revealing of algorithm adjustment possibilities. New movement rules concerning diverse timesteps and disappearing areas may be developed in future work to better fit supermarket scene, which also may be applied in other similar scenes.

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