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NORMALISATION OF MEASURES IN SEGMENT ANALYSIS USING BIOLOGICAL METHODS

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ABSTRACT

In biology, allometric scaling is used to calculate the body condition by comparing measurements of individual organisms to a population average, or baseline, as defined by the allometric equation. A growing number of studies has shown that scaling laws, one of the fundamental features of complex systems, govern many characteristics of urban form. Based on the scaling analogy between organisms and cities, this paper translates the body condition method from biology to urban studies to propose new normalised measures of segment analysis: allometric length, allometric connectivity, allometric angular depth and allometric least angle choice. The study is supported by the segment analysis of street networks in 70 Adriatic and Ionian coastal cities considered in three historical stages. Baselines for the proposed relativized measures are derived from the strong and significant allometry of segment measures. The proposed measures of allometric angular depth and allometric least angle choice are compared against NAIN and NACH (Hillier et al, 2012) resulting in various degrees of compatibility. The proposed allometric normalisation depends on the comparison of a city to a large sample of cities, and thus faces the challenge of dealing with cases that are not part of a regional or typological sample already studied. Based on the results of a previous comparison involving axial maps of a few samples of cities, we speculate that for cities with unknown allometry, adopting the normalisation formulae proposed here would produce more accurate indices than the existing measures by Hillier et al. The proposed method can be used to normalise any segment analysis measure that displays allometry in others scales of the built environment.

KEYWORDS

Normalisation, segment analysis, scaling, allometric subtraction, street network

1. INTRODUCTION

The rendering of size-invariant measures, also known as relativisation or normalisation, is a fundamental precondition for comparative studies that involve built complexes of different size. Segment analysis in space syntax opened the way for enriching the precursor axial map analysis with descriptions of human perception of built space related to angular changes in movement. Normalisation methods for segment analysis introduced a few years ago (Hillier et al, 2012) enabled important characterizations of streets linked to socio-economic aspects of cities including foreground and background networks, and dispersion of centrality. The method was developed upon a conceptual framework centered upon the human conception of distance as affected by change of direction, or angular deviation from the straight line. This led to a mathematical model that modified the least angle segment choice by taking into account angular depth between segments. This paper argues that the normalisation of angular depth and least angle choice in segment analysis by Hillier and his colleagues is problematic in two levels: first, it is based on empirical models that are validated based on correlations with observed patterns of pedestrian and vehicular movement; second, it approaches the normalisation of least angle segment choice by combining choice and depth, rather than relying on choice as an independent variable on its own right. We propose new normalised measures for segment analysis that are based on the allometric scaling of segment measures by translating methods that were originally developed in life sciences in the 1970s. The study is supported by the analysis of a sample of 70 cities considered in three historical stages and analyzed with segment maps. The argument is developed in four main parts: first, we discuss the existing normalisation methods for segment analysis; second, we review allometric scaling and calculation of body condition in life sciences; third we propose normalised measures for segment analysis based on allometry; and finally, we compare the new measures to the existing ones.

2. NORMALISATION OF DEPTH AND CHOICE IN SEGMENT ANALYSIS

The normalisation of segment angular depth NAIN is built upon an empirical model that is aimed at capturing the manner in which angular depth is structured in real city examples and is validated based on high correlations with observed patterns of pedestrian and vehicular movement. Methods that use fixed yardsticks for normalisation are problematic since they rely on observations that might be quite different from other samples to be analyzed in the future. The measure of choice was part of the early space syntax toolbox (Hillier et al., 1987). Although it is based on its own original conceptualization of a spatial network as a justified graph, it produces the same results as the measure of betweenness centrality, which quantifies the number of times a node is used as a bridge in the shortest-path travels from each node to all others in the network (Freeman, 1977). The approach for the normalisation of least angle choice (Hillier et al, 2012) considers the measures of choice and depth as coupled. This is based on the need to improve the power of spatial analysis as a predictor of pedestrian and vehicular movement in the city. Therefore, the normalisation of least angle choice was framed as a problem solving of the so-called choice paradox, where the amount of choice for a segment is related to the integration of the segment, while the total choice of the network is related to its segregation. The mathematical model for normalising choice is then developed in a cost-benefit way related to the human perception of angular depth, i.e. amount of deviation from the linear during travel in a spatial complex. We argue that such approach raises two additional issues: First, the normalisation of choice is dependent on the measure of depth; hence, the current normalisation of choice is indeed a normalisation of the combined choice-depth product. Second, similar to the earlier discussion on depth, such normalisation has been validated based on the strength of predicting levels of movement, and is therefore dependent upon another external measure. Is it possible to normalise choice as an independent variable without relying on angular depth or levels of pedestrian and vehicular movement? The paper tackles this question by translating normalisation methods originally developed in biology.

3. ALLOMETRIC SCALING AND BODY CONDITION IN LIFE SCIENCES

Allometry describes the effect of size on the proportion of measures in an organism during growth (Huxley, 1932; Kleiber, 1932; von Bertalanffy, 1951). Allometry is expressed according to two equivalent equations

$$Y = Y_0 N^{\beta} \qquad \text{or} \qquad lnY = lnY_0 + \beta lnN \qquad (1)$$

where β is the scaling exponent or allometric slope, and Y_0 a constant. Two variables have an allometric relationship when they display a strong linear correlation in a log-log plot.

The calculation of body condition is one of the main applications of allometric scaling in life sciences. Body condition, or condition index, results as the concept of animal's health state by removing the size effect from the weight-length coefficient, or ponderal index (Rohrer, 1921; Thompson, 1961). Body condition is usually estimated according to the residual index method by comparing residual distances of observed points from the predicted points that lie on the line of least-square linear regression of body mass against body length, where the data are usually log-transformed (Gould, 1975; Harvey and Pagel, 1991)

$$ln\hat{\varepsilon} = lnY - lnY' \qquad \text{or} \qquad \hat{\varepsilon} = \exp(lnY - lnY') \qquad (2)$$

where Y is the actual value, Y' is the predicted value, and $\hat{\varepsilon}$ is the residual.

4. ALLOMETRY IN SEGMENT ANALYSIS MEASURES

Allometry and scaling in general have expanded from biology to many other branches of sciences including urban studies, where first discoveries included the scalar relationship between urban population and city size (Naroll and von Bertalanffy, 1956; Stewart, 1947). Ranko Bon pioneered the allometric study of formal aspects of the built environment with findings on the scaling of metric length in circulation networks in buildings and road networks in islands (Bon, 1973; Bon, 1979; Steadman, 2006), during his work with the Philomorphs, an interdisciplinary seminar directed by Arthur Loeb at the Harvard in the '70s. Recent interdisciplinary research in urban studies, geography, and network theory has shown that just like organisms and ecosystems cities conform to scaling laws since they are based on emergent bottom-up processes of social and spatial networks, (Barthélemy 2013; Batty, 2008; Bettencourt and West, 2010; Makse et al., 1995). Allometric scaling is shown to apply to topological measures of street networks (Jiang, 2007; Jiang and Claramunt, 2004) and metric and syntactic measures of axial maps (Shpuza, 2014; Shpuza, 2017). This study is based on the allometric analysis of segment maps of a sample of 70 coastal cities in the Adriatic and Ionian littoral region (hereafter referred to as AI) considered in three historical stages from 19th century, WW2 and 2002-2010 (referred to as S1, S2 and S3) (figure 1), previously analyzed with axial maps. The cities include a wide variety of street patterns, which reflect various socio-political and physiographic influences (Shpuza, 2007). Street networks are defined according to the contiguity of built form and disregarding administrative boundaries (Shpuza, 2011). Segment maps are generated from manually drawn axial maps and analyzed with UCL Depthmap software (Turner, 2010; Varoudis, 2016) by removing stubs of up to 40% in length.

Four allometric relationships are examined from the log-log plots for the segment measures of AI sample: 1) overall length L (not shown in table) to network size N; 2) overall connectivity C to N; 3) overall angular depth D to N; and 4) overall least angle choice B to N (table 1). Overall values are calculated as products of mean values and network size.

The scaling exponents β (1.152, 0.993, 2.271, 2.573) for allometric relationships of length, connectivity, angular depth and least angle choice for AI segment map analysis (table 2) differ slightly to length, connectivity, depth and choice (1.086, 0.968, 2.360 and 2.408) reported for the same sample analyzed with axial maps (Shpuza, 2017). Compared to axial analysis, super-linear scaling of segment length and least angle choice is further increased; super-linear scaling of angular depth decreases; whereas segment connectivity becomes practically linear.

Figure 1 - Map of 70 selected towns and cities along the Adriatic and Ionian coastline

	City	N			$\bar{C}$			$\bar{D}$			$\bar{B}$		
	Country	1	2	1	2	1	2	1	2	3	1	2	3
	GREECE												
1	Koroni	400	503	670	3.9	4.0	3.9	1953	1808	3117	4655	4960	8752
2	Methoni	385	482	707	4.0	3.8	3.7	1933	1887	3052	4646	5086	8252
3	Pylos	254	546	823	3.8	3.9	3.9	905	2127	3629	2716	5977	13687
4	Zakynthos	1000	998	1535	4.0	4.0	4.0	4223	4220	6601	20245	20283	38498
5	Patras	1009	2931	14696	4.6	4.4	4.0	2846	11953	116083	12738	75340	1074060
6	Aigio	403	982	3950	3.9	4.2	3.8	1370	3285	23926	3451	14818	146363
7	Korinthos	166	613	2061	5.1	4.4	4.0	268	2020	9420	479	7387	48832
8	Nafpaktos	310	712	1892	3.7	3.9	3.8	1820	3676	9886	2983	10184	58634
9	Messolonghi	559	852	2240	4.1	4.3	4.2	1684	2428	8092	6002	12735	56069
10	Lefkada	760	906	1714	4.1	4.2	4.1	2519	2894	6421	11871	16097	39163
11	Vonitsa	393	554	1133	3.8	3.9	4.0	1497	2038	4114	4842	7229	18868
12	Preveza	493	890	2264	3.8	3.9	3.9	1957	4071	10927	6189	14376	59112
13	Kerkyra	1506	2229	6522	3.9	3.9	3.5	7174	11932	51943	27972	54529	401697
	ALBANIA												
14	Sarandë	132	257	4624	3.8	3.7	3.5	306	659	50591	665	2645	238747
15	Vlorë	432	1213	10938	3.7	3.7	3.5	1553	5147	94235	4771	24836	735455
16	Durrës	296	1484	12958	3.6	3.7	3.5	932	6488	97596	2582	30735	1093680
	MONTENEGRO												
17	Budva	186	243	2352	3.8	3.8	3.6	522	730	13484	1154	1684	65647
18	Kotor	275	354	1349	3.7	3.6	3.3	1146	1795	10323	2165	3547	40692
19	Herceg Novi	203	579	2943	3.5	3.4	3.4	833	3159	27790	1566	9760	125087
	CROATIA												
20	Dubrovnik	948	2097	3217	3.7	3.5	3.5	4940	15237	23559	16755	67981	112435
21	Korčula	334	454	793	3.8	3.9	3.7	1508	1810	4923	4166	7038	17962
22	Hvar	444	656	1150	3.9	3.8	3.6	1945	3657	8170	3566	7984	21467
23	Stari Grad, Hvar	438	699	1139	3.8	3.7	3.7	1751	3542	7857	5570	9844	27495
24	Makarska	246	353	1779	3.9	3.9	3.6	679	1121	8157	1680	3034	47116
25	Omiš	103	176	1015	3.3	3.4	3.3	294	566	5828	550	1356	37626
26	Split	1552	2436	13097	3.7	3.9	3.6	9065	12187	104253	29607	56249	643955
27	Šibenik	758	1282	2871	3.8	3.8	3.7	3809	6293	19493	9682	21511	82935
28	Biograd na Moru	143	291	2029	3.5	3.7	3.8	546	943	9171	664	2294	58005
29	Zadar	540	1217	8581	4.0	3.8	3.5	1761	5928	69208	5812	25146	460275
30	Senj	375	525	1367	3.7	3.6	3.4	1430	2267	8910	3571	5964	27772
31	Rijeka-Kastav	501	4793	22466	4.2	3.7	3.4	1446	37188	379178	5154	233618	2266420
32	Opatija-Volosko	589	781	1780	3.3	3.4	3.2	3223	4315	15004	6886	9656	47431
33	Pula	263	1816	4333	3.9	4.2	3.7	699	7728	28371	1883	36023	127332
34	Rovinj	619	759	2255	3.7	3.7	3.6	3239	4335	17517	8541	9954	59428
35	Poreč	184	370	1751	3.8	4.0	3.6	432	1004	8864	1046	3790	36105
36	Umag	127	232	1574	4.2	3.9	3.7	233	625	7282	573	1806	33550

Table 1. (first part) - Catalogue of seventy towns and cities along the Adriatic and Ionian coast (listed counterclockwise from Peloponnese to Sicily) over three historical stages (1, 2 and 3). Segment analysis measures: number of segments N, mean connectivity $\bar{C}$, mean angular depth $\bar{D}$, and mean least angle choice $\bar{B}$.

	City Country	N			$\bar{C}$			$\bar{D}$			$\bar{B}$		
		1	2	3	1	2	3	1	2	3	1	2	3
	SLOVENIA												
37	Piran	575	779	921	3.7	3.9	3.9	2279	3013	4083	8300	12129	17021
38	Izola	372	656	1869	3.8	4.0	3.7	1123	2274	9637	3882	8930	40219
39	Koper	937	999	4122	3.9	4.0	3.7	3589	3915	29128	15266	17040	137519
	ITALY												
40	Muggia	188	390	1241	3.6	3.6	3.3	557	1397	8427	1233	4327	31756
41	Trieste	1433	6826	14855	4.2	3.9	3.6	6413	50282	158701	28047	296451	915592
42	Monfalcone	231	1275	5141	3.7	3.8	3.7	669	5223	29806	1792	24420	249158
43	Grado	112	466	1057	3.7	4.5	4.2	316	1419	4406	396	4138	19332
44	Venezia	6866	7478	7455	3.5	3.7	3.6	147026	110949	110410	530036	449060	446634
45	Chioggia	552	1649	5890	3.8	4.0	3.7	1913	7170	35017	8184	46037	335068
46	Cesenatico	93	442	2259	3.9	4.2	3.9	217	1330	14596	254	4651	65867
47	Pesaro	586	1733	6961	4.0	4.0	3.8	1677	6629	49194	7089	33251	321145
48	Fano	443	1043	3876	4.0	4.2	3.8	1147	3046	20436	4514	18048	141281
49	Ancona	1014	2210	9361	3.7	3.8	3.6	5051	13070	80775	20941	52248	459770
50	San Benedetto	321	884	3009	4.4	4.4	4.0	935	2680	13472	2299	13409	97439
51	Ortona	321	667	1476	4.4	4.2	4.1	818	2186	6404	3054	8722	37420
52	Termoli	166	533	3437	3.8	4.0	3.5	475	2146	27770	1068	6796	117013
53	Manfredonia	183	401	3666	4.6	4.4	4.2	324	958	18339	751	3477	143111
54	Barletta	614	1101	3426	4.2	4.3	3.9	1933	3581	17402	8859	18401	126257
55	Trani	674	1414	5043	4.0	4.3	3.8	2419	4895	22833	8940	25680	252415
56	Molfetta	532	1276	3567	4.4	4.6	4.3	1625	4048	14506	5771	20771	114484
57	Bari	908	2360	7183	4.0	4.3	4.0	4193	10753	39447	13496	59359	323585
58	Monopoli	631	1047	3967	3.7	4.0	3.9	3064	5045	20946	9307	17051	130792
59	Brindisi	802	1973	5527	4.1	4.0	3.9	2933	11326	36372	10540	51621	219470
60	Otranto	194	351	1074	3.7	3.7	3.6	615	1367	7450	1177	2835	20938
61	Gallipoli	486	936	1867	3.7	4.1	4.0	1972	4703	10156	5597	17624	53672
62	Taranto	982	1997	5282	4.0	4.2	3.9	3353	11898	39839	23567	61361	245376
63	Crotone	804	1313	2237	4.1	4.3	4.1	3525	5679	10149	9497	20245	45261
64	Reggio Calabria	469	2355	7108	4.2	4.3	3.7	1432	10086	51850	4599	67313	351746
65	Messina	1978	4077	14571	4.5	4.3	3.6	6577	18804	146921	44075	143056	1027130
66	Acireale	1293	1673	4913	3.8	3.7	3.6	7045	9558	33912	26172	39479	204526
67	Catania	2273	5718	22114	4.2	4.2	3.7	8763	27433	223412	58353	240609	1797320
68	Augusta	225	448	1245	4.0	3.8	4.0	497	1368	5641	1671	5479	24593
69	Siracusa	787	1549	6339	3.7	4.2	3.8	3082	7231	47732	11781	36237	281634
70	Avola	779	1821	4152	4.0	4.4	4.3	2086	5945	15889	13528	40536	147028
	Min	93	176	670	3.3	3.4	3.2	217	566	3052	254	1356	8252
	Max	6866	7478	22466	5.1	4.6	4.3	147026	110949	379178	530036	449060	2266420
	Mean	659.3	1372.9	4611.1	3.9	4.0	3.8	4316	7750	37743	16020	38518	247431

Table 1 (second part)

Plot	Linear Fit Equation	R²	Allometric Equation
$\ln L$ vs $\ln N$	y=1.152x+2.291	0.941	$L = 9.885N^{1.152}$
$\ln C$ vs $\ln N$	y=0.993x+1.403	0.996	$C = 4.067N^{0.993}$
$\ln D$ vs $\ln N$	y=2.271x-0.394	0.991	$D = 0.674N^{2.271}$
$\ln B$ vs $\ln N$	y=2.573x-1.127	0.997	$B = 0.324N^{2.573}$

Table 2 - Statistics of log-log plots and the resulting allometric equations of L, C, D and B versus magnitude N calculated for the sample of 70 Adriatic and Ionian coastal cities in three historical stages (210 cases) analyzed with segment maps. All logarithmic measures presented in the paper are in natural base, and all correlations have significance at $p<0.0001$.

5. NORMALISED MEASURES OF SEGMENT ANALYSIS BASED ON ALLOMETRIC SUBTRACTION

This study follows the long tradition of formulating descriptive models in architectural morphology by translating descriptive models from biology (March and Steadman, 1971; Steadman, 1979; Steadman, 1983; Steadman and Mitchell, 2010). The paper proposes normalised allometric measures Y_a^X for segment analysis by applying the allometric subtraction model recently developed for normalisation of measures in axial analysis (Shpuza, 2017).

$$Y_a^X = \exp(lnY - \ln Y') \qquad (3)$$

The subscript 'a' denotes allometry, while the superscript 'X' denotes the measure of size used for deriving the allometric relationship. The allometric measure is formulated analogously to body condition of an organism (eq. 2), by comparing the observed value Y for the city against the predicted value Y', i.e. the allometric average condition for the sample of cities. The latter is calculated from the allometric equation between Y and the size variable X.

We define four allometric measures based on the criterion of allometric subtraction in order to quantify in a scale-invariant manner segment analysis measures: lengt L_a^N, connectivity C_a^N, angular depth D_a^N, and least angle choice B_a^N (table 3). Values of allometric measures vary about 1 so that values smaller than 1 indicate a condition below the allometric average for the sample, while values greater than 1 indicate a condition above.

Allometric Formula	Min	Max	Mean
$L_a^N = \exp(lnL - \ln(9.885N^{1.152}))$	0.481	3.972	1.058
$C_a^N = \exp(lnC - \ln(4.067N^{0.993}))$	0.834	1.299	1.002
$D_a^N = \exp(lnD - \ln(0.674N^{2.271}))$	0.594	2.899	1.034
$B_a^N = \exp(lnB - \ln(0.674N^{2.271}))$	0.476	2.167	1.015

Table 3 - Measures of allometric length L_a^N, allometric connectivity C_a^N , allometric angular depth D_a^N and allometric least angle choice B_a^N derived from allometric equations (table 2) for segment maps of the sample of 70 Adriatic and Ionian coastal cities in three historical stages (210 cases). Values L, C, D and B are the totals for each city and are calculated with products between means and network size N.

The proposed normalised allometric measure characterizes the entire city. The allometric measure for a network component, e.g. a segment in the segment map, is calculated by proportioning the allometric measure for the city by the ratio between the measure for the component and the city aggregate. For example, the allometric angular depth for a segment d^N_a is calculated as

$$d^N_{a,i} = \frac{d_i}{D} D^N_a \quad \text{or} \quad d^N_{a,i} = \frac{d_i D^N_a}{N\bar{D}} \qquad (4)$$

where D^N_a is the allometric angular depth, D is the aggregate of angular depth d for each segment, d_i is the angular depth of a segment to all other segments, N is the number of segments in the map, and $\bar{D}$ is the mean angular depth. The other allometric measures for network components are calculated similarly.

The comparison between allometric measures and existing segment analysis measures (figure 2) shows mixed results. Strong correlations for length $\bar{L}$ versus L^N_a (R2 = 0.753), connectivity $\bar{C}$ versus C^N_a (R2 = 0.989), and integration $N\bar{A}IN$ versus allometric angular depth D^N_a (R2=0.783) shows that, while not precise, using the existing measures gives good approximations to allometric measures. The non-existent correlation (R2 = 0.074) between least angle choice $N\bar{A}CH$ and allometric least angle B^N_a choice shows the two measures are incompatible. The allometric least angle choice is calculated based on the comparison to values of choice in a large sample of cities, instead of the existing normalised least angle choice NACH, which is a product of both depth and choice.

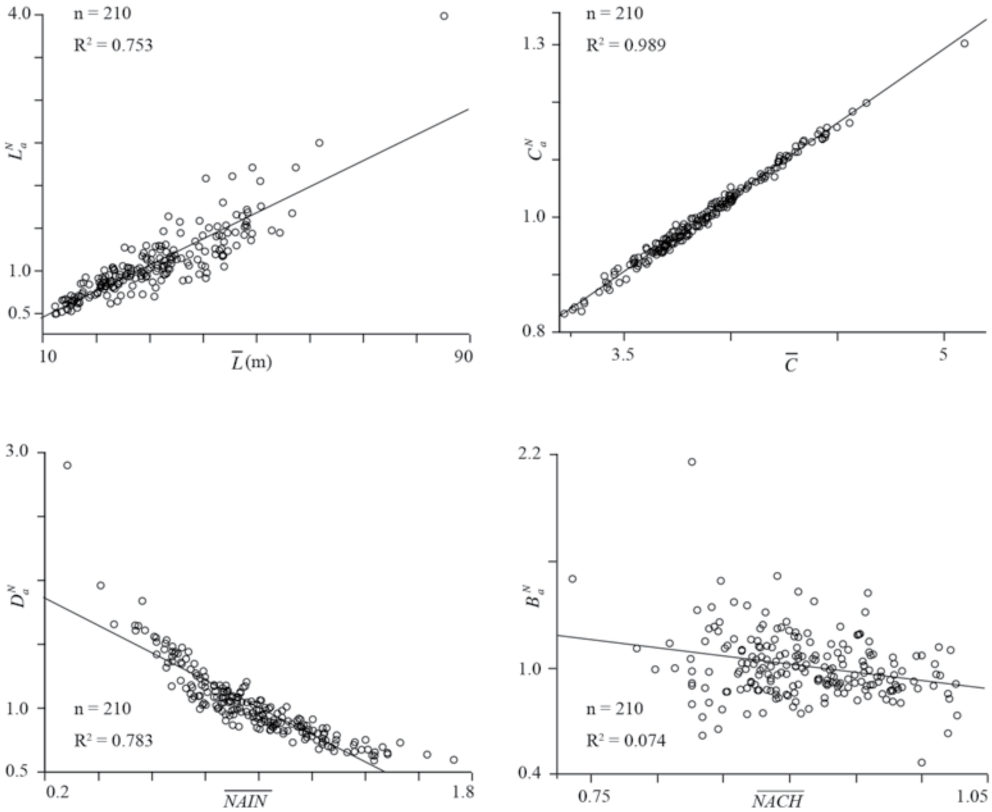

Figure 2 - Comparative plots between the existing measures and the proposed allometric measures for segment maps of 70 Adriatic and Ionian cities in three historical stages, 210 data points: a) mean metric length $\bar{L}$ versus allometric segment length L^N_a; b) mean connectivity $\bar{C}$ versus allometric segment connectivity C^N_a; c) mean normalised angular integration $N\bar{A}IN$ versus allometric angular depth D^N_a; d) mean normalised least angle choice $N\bar{A}CH$ versus allometric least angle choice B^N_a.

6. ALLOMETRIC STAR MODELS FOR COMPARATIVE STUDIES

When the proposed allometric measures for segment analysis are employed for comparative urban studies, the four-pointed star model (Hillier et al., 2012) is modified such that D^N_a, B^N_a, minimum $d^N_{a,i}$ (as an inverse of integration) and maximum $b^N_{a,i}$ replace mean NAIN, mean NACH, maximum angular integration, and maximum least angle choice (figure 3). As a corollary of equation (4), the mean of $d^N_{a,i}$ for all segments in the map equals the allometric measure for the entire city D^N_a, whereas the mean of $b^N_{a,i}$ equals B^N_a. The proposed allometric measures are not only normalised, where the measures are rendered size free for inter-sample comparative studies, they are also calculated based on the allometric curve, which serves as a mean for the sample. Therefore, the vertical axis of the star model constructed with allometric measures does not require standardization with z-scores as in the original version (Hillier et al, 2012). The vertical axis in the four-pointed star is scaled from -1.50 at the center to 1.50 on top and bottom in order to accommodate D^N_a values, which range from 0.594 to 2.899, and B^N_a values, which range from 0.476 to 2.167 (table 3). The variation of D^N_a and B^N_a is a characteristic of the AI sample, thus the scaling of the vertical axis of the star model might differ for other city samples. The $mind(d^N_{a,i})$ values for the AI sample range from 2.632E-05 for Catania-2007 to 0.00783 for Biograd 1826, with a mean of 0.001086 and a standard deviation σ= 0.0013255. The $max(b^N_{a,i})$ values range from 0.000329 for Korinthos-2003 to 0.05697 for Omiš -1834, with a mean of 0.0135289 and σ =0.0095296. For the purpose of star model, the values of $mind(d^N_{a,i})$ and $max(b^N_{a,i})$ values in the horizontal axis are converted into z-scores

$$z = \frac{x - \mu}{\sigma} \qquad (5)$$

where x is the raw score of $mind(d^N_{a,i})$ or $max(b^N_{a,i})$ values, σ is the standard deviation, and μ is the mean. The star model is scaled with $mind(d^N_{a,i})$ z-scores, which range from -0.799 to 5.084 and $max(b^N_{a,i})$ z-scores, which range from -1.385 to 4.56. The $mind(d^N_{a,i})$ outliers include small towns Grado-1825, Cesenatico-1818, Omiš-1834, and Biograd 1826 with z-scores greater than 3. The $max(b^N_{a,i})$ outliers include also small towns of Cesenatico-1818, Umag-1873, Umag-1943, and Omiš-1834 with z-scores greater than 3.

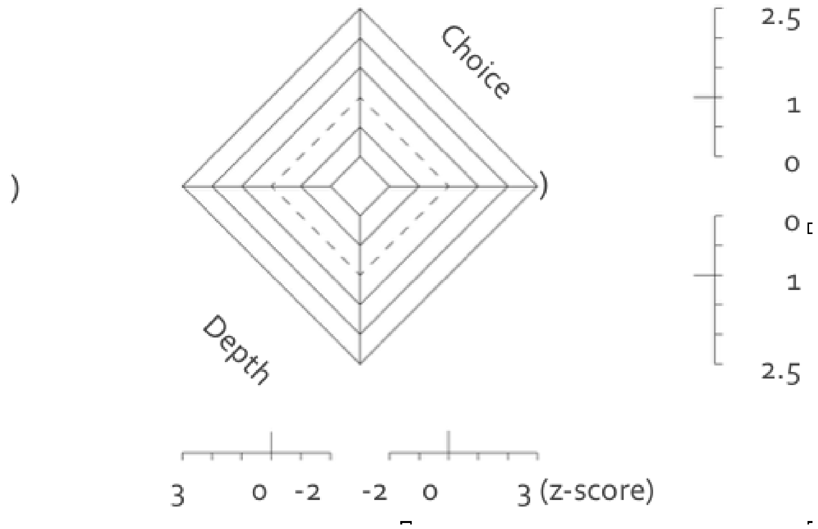

Figure 3 - Base four-pointed star model with normalised allometric measures for segment analysis: top) allometric least angle choice B^N_a; right) highest least angle choice value b^N_a; bottom) allometric angular depth D^N_a; left) lowest angular depth value d^N_a. The 1-0-1-0 value rhombus is shown dashed.

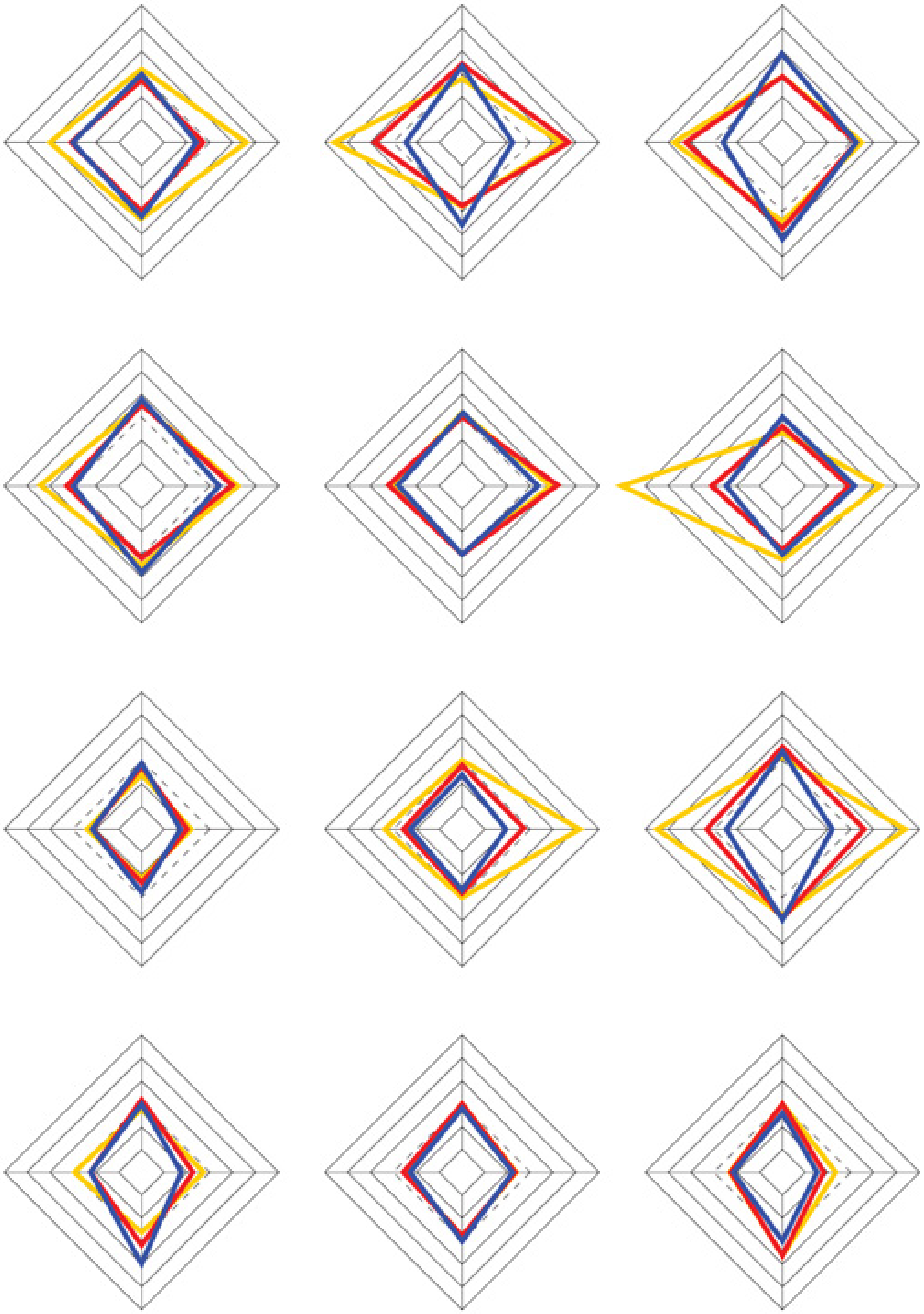

Figure 4 - Star models with normalised segment analysis measures B^N_a, $max(b^N_{a,i})$, D^N_a and $mind(d^N_{a,i})$ for 12 cities in three historical stages, S1 in orange, S2 in red and S3 in blue with smallest networks in upper two rows and largest networks in lower two rows.

We illustrate in star models 12 cities in three historical stages (figure 4), selected as the smallest segment maps in the third historical stage S3: Koroni, Greece 1829-1945-2003; Sarandë, Albania 1925-1945-2006; Kotor, Montenegro 1838-1943-2005; Korčula, Croatia 1836-1940-2003; Piran, Slovenia 1818-1943-2005; and Grado, Italy 1825-1943-2005, and the largest segment maps in S3 in the six countries on the Adriatic and Ionian coastline: Patras, Greece 1894-1943-2007; Durrës, Albania 1876-1943-2007; Herceg Novi, Montenegro 1838-1940-2007; Rijeka, Croatia 1865-1950-2007; Koper, Slovenia 1819-1956-2006; and Catania, Italy 1820-1942-2007. Considering the 36 selected cases (figure 4), small networks display a greater departure from the allometric mean, i.e. the 0-value rhombus in the center of the star model. This is observed both in terms of the comparison between smaller towns and larger cities, as well as between the smaller networks in the first historical period S1 (orange), and the larger networks in the subsequent periods S2 (red) and S3 (blue). The distribution of proposed allometric measures considered separately according to subsamples of 70 cases for three historical stages S1, S2 and S3 results in means around 1, while skewness values are much higher for allometric angular depth during S1 at 2.755 and allometric least angle choice during S3 at 2.308 (table 4).

	B^N_a	$max(b^N_{a,i})$	D^N_a	$mind(d^N_{a,i})$
S1	0.156	1.133	2.755	1.691
S2	0.888	1.370	1.231	1.619
S3	2.308	0.967	0.958	1.416

Table 4 - Skewness of distribution of B^N_a, $max(b^N_{a,i})$, D^N_a and $mind(d^N_{a,i})$ for subsamples of three historical periods S1 (1800-1900), S2 (WW2) and S3 (2002-2010) of the AI towns and cities.

7. CONCLUSIONS

The study proposes normalised measures for segment analysis based on the allometric subtraction, one of the most common methods in biology for quantifying the body condition of an organism. The study is supported by the analysis of a sample of 70 coastal cities in the Adriatic and Ionian region considered in three historical stages. The translation of biological methods for the study of cities is based on the functional analogy between cities and organisms as expressed by the existence of strong and significant allometric scaling of length, connectivity, angular depth and least angle choice in segment map analysis. Allometric measures show various degrees of compatibility with existing normalised segment measures, while there is a complete lack of correspondence between normalised choice NACH and allometric choice. Star models for comparative studies are modified with the new allometric measures; they reveal changes in depth and choice during urban growth. The allometric subtraction produces measures that are tied to a sample of study and therefore face the issue of applying allometric formulae for cities that belong to samples not yet analyzed. While this problem requires further analysis with additional city samples, a previous study of axial maps (Shpuza, 2017) has shown that applying allometric measures generated in other city samples produces more reliable results than using normalised measures that are based on means or theoretical yardsticks. The proposed method can be used to generate normalised measures for segment maps of any scale of the built environment.

REFERENCES

Barthélemy M, Bordin P, Berestycki H, Gribaudi M, 2013, "Self-organization versus top-down planning in the evolution of a city" *Scientific Reports* 3 (2153) 1-7

Batty M, 2008, "Size, scale and shape of cities" *Science* 319 769–771

Bettencourt L M A, West G B, 2010, "A unified theory of urban living" *Nature* 467 (7318) 912-913

Bon R, 1973, "Allometry in the topologic structure of architectural spatial systems" *Ekistics* 36 (215) 270–6

Bon R, 1979, "Allometry in topologic structure of transportation networks" *Quality and Quantity* 13 307–326

Freeman L C, 1977, "A set of measures of centrality based on betweenness" *Sociometry* 40 (1) 35-41

Gould S J, 1975, "Allometry in primates, with emphasis on scaling and evolution of the brain" Ed. F S Szalay, *Approaches to Primate Paleobiology, Contributions to Primatology*, S. Karger, Basel, München, Paris, London, New York, Sydney, 5 244-292

Harvey P H, Pagel M D, 1991, *The Comparative Method in Evolutionary Biology*, Oxford University Press, Oxford

Hillier B, Burdett R, Peponis J, Penn A, 1987, "Creating life: Or, does architecture determine anything?" *Architecture and Behaviour* 3 (3) 233-250

Hillier B, Yang T, Turner A, 2012, "Normalising least angle choice in Depthmap and how it opens up new perspectives on the global and local analysis of city space" *Journal of Space Syntax* 3 (2) 155-193

Huxley J S, 1932, *Problems of Relative Growth*, Methuen, London

Jiang B, 2007, "A topological pattern of urban street networks: Universality and peculiarity" *Physica* A 384 647–655

Jiang B, Claramunt C, 2004, "Topological analysis of urban street networks" *Environment and Planning B: Planning and Design* 31 (1) 151–162

Kleiber M, 1932, "Body size and metabolism" *Hilgardia: A Journal of Agricultural Science* 6 (11) 315-353

Makse H A, Havlin S, Stanley H E, 1995, "Modelling urban growth patterns" *Nature* 377 608–612

March L, Steadman P, 1971, *The Geometry of Environment: An Introduction to Spatial Organization and Design*, RIBA Publications, London

Naroll R S, von Bertalanffy L, 1956, "The principle of allometry in biology and social sciences" *General Systems* 1 76-89

Rohrer F, 1921, "Der Index der Körperfülle als Maß des Ernährungszustandes (The index of corpulence as measure of nutritional state)" *Münchener Medizinische Wochenschrift* 68 580–582.

Shpuza E, 2007, "Evolution of Street Networks in Adriatic and Ionian Coastal Cities 1769-2007" in Eds. D Koch, L Marcus and J Steen Proceeding of the *7th International Space Syntax Symposium* 101:1-101:15

Shpuza E, 2011, "A coupled configurational description of boundary shape based on distance and directionality" *Artificial Intelligence for Engineering Design, Analysis and Manufacturing* 25 357–374

Shpuza E, 2014, "Allometry in the syntax of street networks: evolution of Adriatic and Ionian coastal cities 1800-2010" *Environment and Planning B: Planning and Design* 41 (3) 450-471

Shpuza E, 2017, "Relative size measure of urban form based on allometric subtraction" *Environment and Planning B: Urban Analytics and City Science* 44 (1) 141-159

Steadman P, 1979, *The Evolution of Designs: Biological Analogy in Architecture and the Applied Arts*, Cambridge University Press, Cambridge

Steadman P, 1983, *Architectural Morphology: An Introduction to the Geometry of Building Plans*, Pion, London

Steadman P, 2006, "Allometry and built form: revisiting Ranko Bon's work with the Harvard Philomorphs" *Construction Management and Economics* 24 (7) 755-765

Steadman P, Mitchell L, 2010, "Architectural morphospace: mapping worlds of built form" *Environment and Planning B: Planning and Design*, 37 (2) 197-220

Stewart J Q, 1947, "Empirical mathematical rules concerning the distribution and equilibrium of population" *Geographical Review* 37 (3) 461-485

Teklenburg J A F, Timmermans H J P, van Wagenburg A F, 1993, "Space syntax: standardized measures and some simulations" *Environment and Planning B: Planning and Design* 20 (3) 347-357

Thompson D'A W, 1961, *On Growth and Form*, Cambridge University Press, Cambridge; abridged by J T Bonner from the 1917 edition

Turner A, 2010, UCL *Depthmap: Spatial Network Analysis Software* v. 8.04.18b (University College London, London)

Varoudis T, 2016, *DepthmapX* (University College London, London)

von Bertalanffy L, 1951, *Theoretische Biologie Vol. II*, Stoffwechsel, Wachstum, 2nd Edition, Francke, Bern